NAMD Tutorial (Part 2)

- **2 Analysis**
  - 2.1 Equilibrium
    - 2.1.1 RMSD for individual residues
    - 2.1.2 Maxwell-Boltzmann Distribution
    - 2.1.3 Energies
    - 2.1.4 Temperature distribution
    - 2.1.5 Specific Heat
  - 2.2 Non-equilibrium properties of protein
    - 2.2.1 Heat Diffusion
    - **2.2.2 Temperature echoes**

Main funding:
National Center for Research Resources

Temperature Echoes in Proteins

- Coherent motion in proteins: Echoes
- Generation of echoes in *ubiquitin* via velocity reassignments
  1) Temperature quench echoes
  2) Constant velocity reassignment echoes
  3) Velocity reassignment echoes

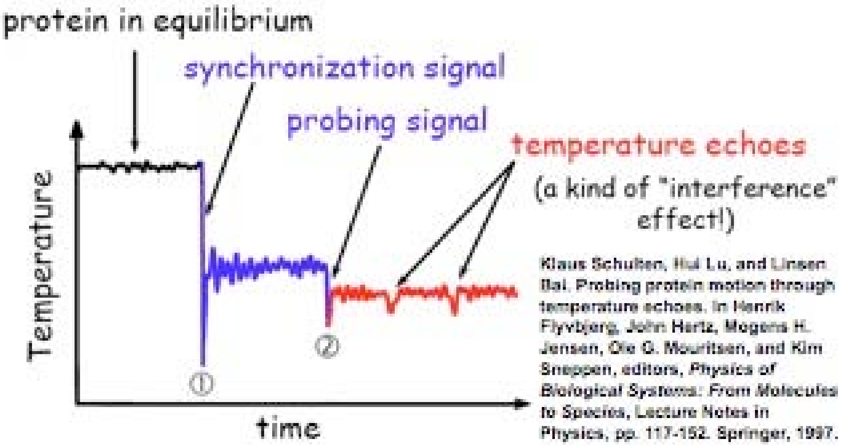
temperature ⇔ velocities

kinetic temperature:

$$T(t) = \frac{2}{(3N-6)k_B} \sum_{n=1}^{3N-6} \frac{m_n v_n^2(t)}{2}$$

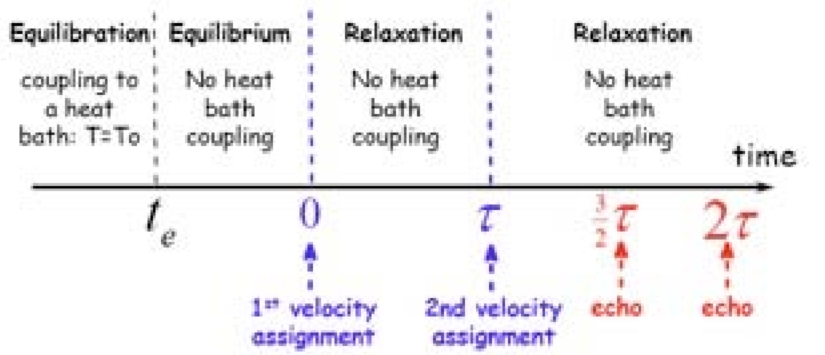
Temperature Echoes

- are sharp, resonance-like features in the time evolution of the protein's temperature
- can be produced through 2 consecutive velocity reassignments



Klaus Schulten, Hui Lu, and Linsen Bai. Probing protein motion through temperature echoes. In Henrik Flyvbjerg, John Hertz, Mogens H. Jensen, Ole G. Mouritsen, and Kim Sneppen, editors, *Physics of Biological Systems: From Molecules to Species*, Lecture Notes in Physics, pp. 117-152. Springer, 1997.

Producing Temperature Echoes by Velocity Reassignments in Proteins



Temperature quench echoes: $v_i(0) = v_i(\tau) = 0$

Const velocity reassignment echoes: $v_i(0) = v_i(\tau) = u_i$

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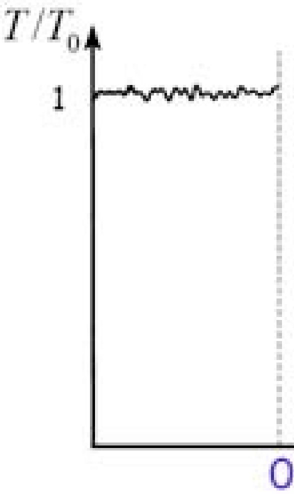
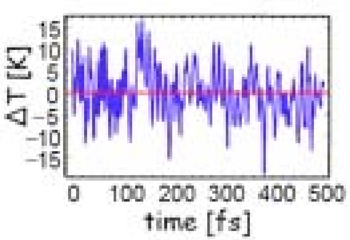
Generating T-Quench Echo: Step1

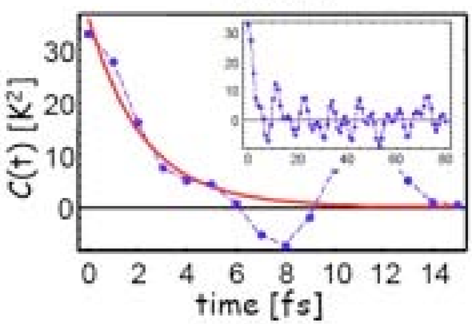


- your system is ubiquitin (1UBQ) in vacuum, pre-equilibrated at T_0=300K
- run all simulations in the *microcanonical* (NVE) ensemble
- psf, pdb and starting binary coordinate and velocity files are available in "common/"
- use NAMD2 configuration file "equil.conf" located in "01_equil_NVE/" to complete a 500 fs (# simulation steps) run
- extract the temperature time series T(t) from the NAMD2 log (output) file
- plot T(t)
- calculate: $\langle T \rangle$, $\sqrt{\langle T^2 \rangle}$, $C_{TT} = \langle \delta T(t) \delta T(0) \rangle$

Temperature Autocorrelation Function

$$\Delta T(t) = T(t) - \langle T(t) \rangle$$





$$C(t) = \langle \Delta T(t) \Delta T(0) \rangle$$

$$\rightarrow C(t_i) \approx \frac{1}{N-i} \sum_{n=1}^{N-i} \Delta T(t_{n+i}) \Delta T(t_n)$$

$$C(t) = C(0) \exp(-t/\tau_0)$$

Temperature relaxation time:

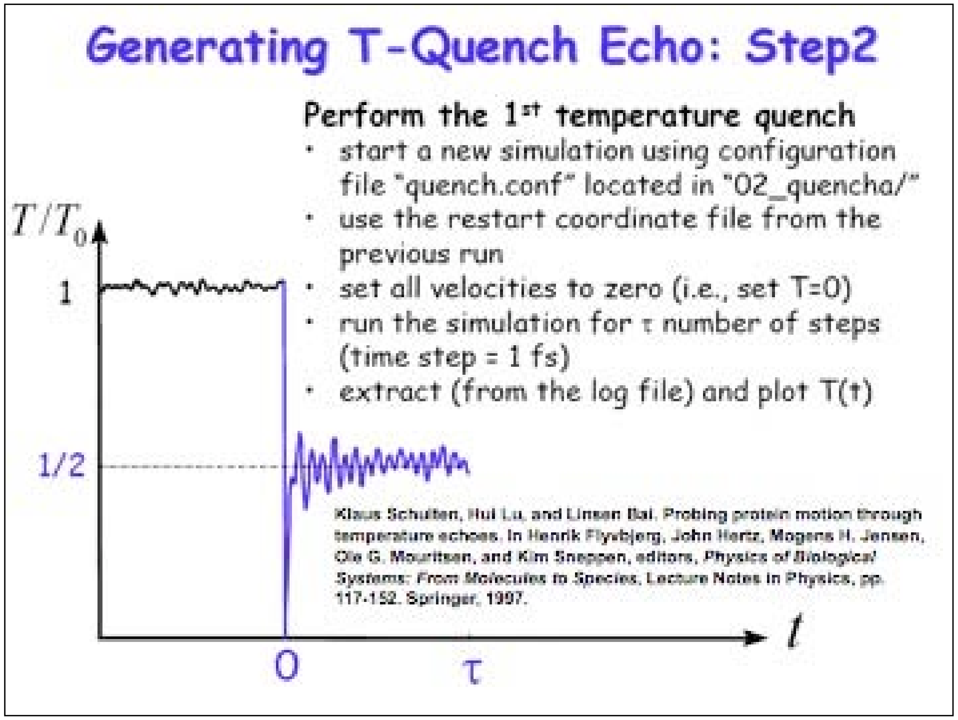
$$\tau_0 \approx 2.2\, fs$$

Mean temperature:

$$\langle T \rangle = 299\, K$$

RMS temperature:

$$\sqrt{\langle \Delta T^2 \rangle} = \sqrt{C(0)} = 6\, K$$


Generating T-Quench Echo: Step2
Perform the 1st temperature quench
• start a new simulation using configuration file "quench.conf" located in "02_quencha/"
• use the restart coordinate file from the previous run
• set all velocities to zero (i.e., set T=0)
• run the simulation for τ number of steps (time step = 1 fs)
• extract (from the log file) and plot T(t)
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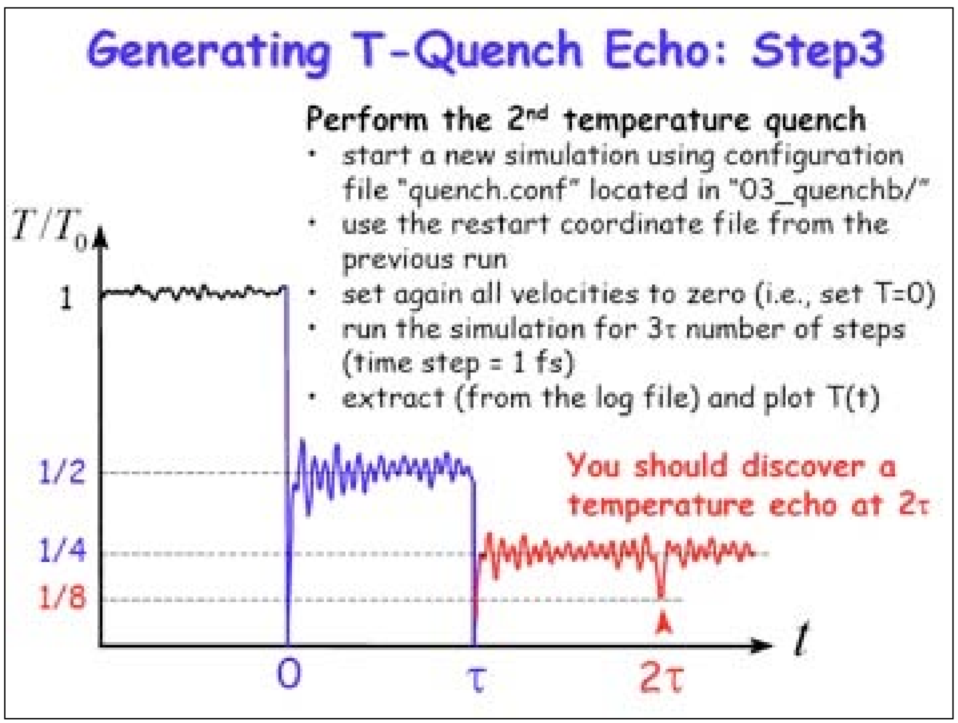
T/T0
1
1/2
0
τ
t


Generating T-Quench Echo: Step3
Perform the 2nd temperature quench
• start a new simulation using configuration file "quench.conf" located in "03_quenchb/"
• use the restart coordinate file from the previous run
• set again all velocities to zero (i.e., set T=0)
• run the simulation for 3τ number of steps (time step = 1 fs)
• extract (from the log file) and plot T(t)
You should discover a temperature echo at 2τ
T/T0
1
1/2
1/4
1/8
0
τ
2τ
t

Explanation of the T-Quench Echo

Assumption: protein ≈ collection of weakly interacting harmonic oscillators with dispersion $\omega = \omega_\alpha$, $\alpha = 1, \ldots, 3N-6$

Step1: $t < 0$

$$x(t) = A_0 \cos(\omega t + \theta_0)$$
$$v(t) = -\omega A_0 \sin(\omega t + \theta_0)$$

Step2: $0 < t < \tau$

$$\left.\begin{aligned} x_1(t) &= A_1 \cos(\omega t + \theta_1) \\ v_1(t) &= -\omega A_1 \sin(\omega t + \theta_1) \end{aligned}\right\} \xrightarrow{v_1(0)=0} \begin{cases} A_1 = A_0 \cos\theta_0 \\ \theta_1 = 0 \end{cases}$$

Step3: $t > \tau$

$$\left.\begin{aligned} x_2(t) &= A_2 \cos(\omega t + \theta_2) \\ v_2(t) &= -\omega A_2 \sin(\omega t + \theta_2) \end{aligned}\right\} \xrightarrow{v_2(\tau)=0} \begin{cases} A_2 = A_1 \cos\omega\tau \\ \theta_2 = -\omega\tau \end{cases}$$

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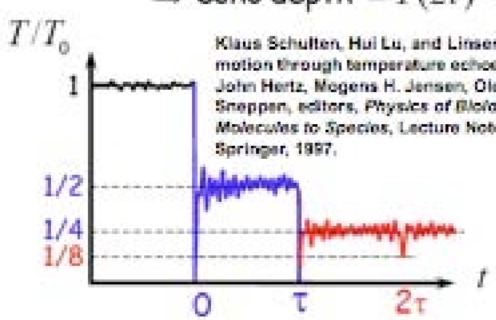
T-Quench Echo: Harmonic Approximation

$$T(t) \approx \frac{T_0}{4}\left[1 - \langle\cos(2\omega(t-\tau))\rangle - \frac{1}{2}\langle\cos(2\omega(t-2\tau))\rangle\right]$$

$$\approx \begin{cases} 0 & \textit{for } t = \tau \\ T_0/8 & \textit{for } t = 2\tau \\ T_0/4 & \textit{otherwise} \end{cases}$$

$\Rightarrow$ echo depth $= T(2\tau) - T_0/4 = T_0/8$



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$T(t)$ and $C_{TT}(t)$

It can be shown:

$$\langle \cos(2\omega t) \rangle = \frac{\langle \delta T(t)\, \delta T(0) \rangle}{\langle \Delta T^2 \rangle} = C_{TT}(t), \qquad \delta T(t) = T(t) - \langle T \rangle$$

Accordingly,

$$T(t) \approx \frac{T_0}{4}\left[1 - \langle \cos(2\omega(t-\tau)) \rangle - \frac{1}{2} \langle \cos(2\omega(t-2\tau)) \rangle \right]$$

$$\downarrow$$

$$= \frac{T_0}{4}\left[1 - C_{TT}(t-\tau) - \frac{1}{2} C_{TT}(t-2\tau) \right]$$

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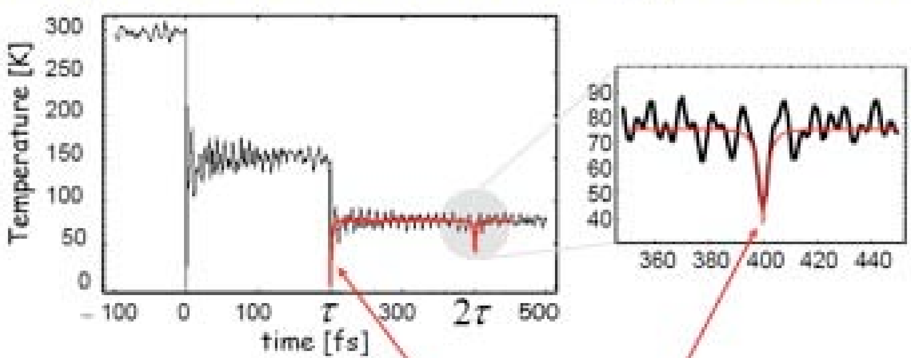
T-Quench Echo: Harmonic Approximation



$$T(t) \approx \frac{T_0}{2}\left(1 - C_{TT}(t-\tau) - \frac{1}{2} C_{TT}(|t-2\tau|)\right)$$

$$C_{TT}(t) = \exp(-t/\tau_0), \qquad \tau_0 \approx 2.2\ fs$$

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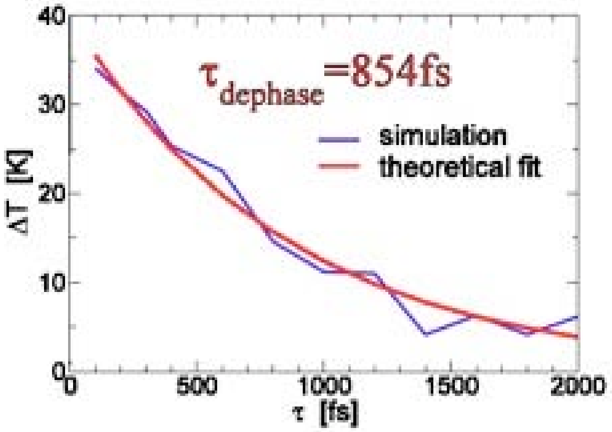
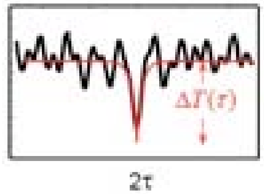
Dephasing Time of T-Quench Echoes





$$\Delta T(\tau) = \Delta T(0)\exp[-\tau / \tau_{dephase}]$$

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Constant Velocity Reassignment Echo ?

Can we get temperature echo(es) by reassigning the same set of atomic velocities (corresponding to T_0!) at $t = 0$ and $t = \tau$?

$$v_i(0^+) = v_i(\tau^+) = u_i,\ \ i = 1,\dots,3N-6$$