# Feynman Rules

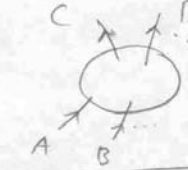
Goal is to determine for initial (i) and final (f) states the matrix elements

$$S_{fi} = \delta_{fi} + R_{fi}$$

$R_{fi}$ can be written, in general,

$$R_{fi} = i\,(2\pi)^4\, \delta(p_i - p_f)\, T_{fi}$$

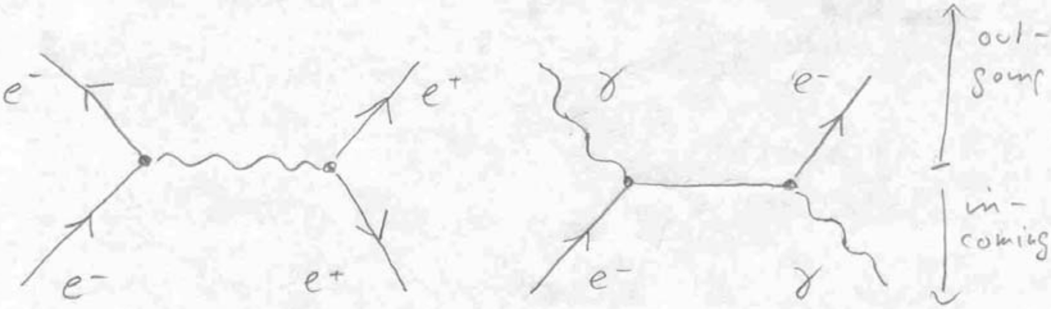
where $p_i$ and $p_f$ are the sum of initial and final momenta. Hence, usually $T_{fi}$ is calculated. The rules below refer to $R_{fi}$.




$A, B, C, \dots$ leptons or photons

## Rule 1: Diagrams

For a given process $A + B + \dots \to C + D + \dots$ draw all connected diagrams of order $n$ (= number of vertices). Exterior and interior lepton lines are shown as directed lines such that the arrows indicate the flow of negative charge. (see examples)




For a particle ($e^-$, $\mu^-$, $\tau^-$), the arrow points to the vertex when it is incoming, away from it, when it is outgoing; for an antiparticle ($e^+$, $\mu^+$, $\tau^+$), it is the opposite.

The following factors (operators) are written right to left following the direction of arrows.

## Rule 2: Outer lines

Let $f$ be $e$, $\mu$, or $\tau$. Outer lines are represented by

Incoming $f^-$ : $u_f^{(r)}(p)$

Outgoing $f^-$ : $u_f^{(r)}(p)$

Incoming $f^+$ : $\bar{v}_f^{(r)}(p)$

Outgoing $f^+$ : $v_f^{(r)}(p)$

where $u$ $(v)$ are the Dirac spinors representing free positive (negative) energy free particle solutions; $r$ denotes the spin orientation and $p$ the four momentum.

Incoming or outgoing photon : $\varepsilon_\mu^{(\lambda)}(p)$

where $\lambda = 1, 2$ denotes the photon polarization and $p$ its momentum. $\varepsilon_\mu^{(\lambda)}$ is contracted with the $\gamma^\mu$ of vertices (next rule) $\varepsilon_\mu^{(\lambda)} \gamma^\mu = \not{\varepsilon}^{(\lambda)}$

## Rule 3 : Vertices

For each vertex assign an $e\gamma^\mu$ as well as a $\delta$-function for the linked 4-vector momenta (three for each vertex) such that 4-momentum is conserved at each vertex. The sign of the momenta is positive/negative depending on wether particles are incoming/outgoing.

## Rule 4 : Inner lines

Inner lepton lines are presented by

$$\frac{\not{p} + m_f \mathbb{1}}{p^2 - m_f^2 + i\varepsilon} \qquad \varepsilon \to 0$$

where $\not{p} = p_\mu \gamma^\mu$. Inner photon lines are represented by

$$\frac{-g_{\mu\nu}}{k^2 + i\varepsilon} \qquad \varepsilon \to 0$$

The indices $\mu, \nu$ are contracted with the $e\gamma^\mu$ and $e\gamma^\nu$ operators representing the vertices connected to the photon line.

Rule 5: Integration

The expressions resulting from rules 3–4 are to be integrated over the momenta of all inner lines. After this integration there remains an overall $\delta$-function factor $\delta(p_i - p_f)$

Rule 6: Sign

a) Assign a factor $(-1)^\pi$ where $\pi$ is determined by permutations between leptons of equal kind

$$\pi = \begin{cases} 0 & \text{permutation even} \\ 1 & \text{permutation odd} \end{cases}$$

b) Assign a second factor $(-1)^L$ where $L$ is the number of closed lepton loops.

Rule 7: C-invariance

When an odd number of photons attach to a closed loop of inner lepton lines the diagram vanishes

Rule 8: Numerical factors

Let $l_a$ = number of external lepton lines
$l_i$ = " " internal " "
$b_a$ = " " external photon "
$b_i$ = " " internal " "

Assign the factor

$$\left[(2\pi)^{-3/2}\right]^{l_a+b_a} i^{n+l_i+b_i}\,(2\pi)^{4(n-l_i-b_i)}$$