Physics 481: Quantum Mechanics II
Midterm Exam

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March 29, 2000

Problem 1: Two Spin-$\frac{1}{2}$ Particles

(a) Two spin-$\frac{1}{2}$ particles in an external static magnetic field $B_o \hat{e}_3$ are described by the Hamiltonian

$$H \;=\; J\, \vec{S}^{(1)} \cdot \vec{S}^{(2)} \;+\; g\, \mu_B B_o \left(S_3^{(1)} \;+\; S_3^{(2)} \right) \tag{1}$$

where g and μ_B are well-known physical constants. Determine the stationary states and the respective energies of the system. Sketch the energies as a function of B_o.

(b) Apply a weak, time-dependent magnetic field $B(t) = b_o \cos(\omega t)\, \hat{e}_1$. What values of ω must be chosen to induce transitions between the eigenstates in (a).

(c) For properly chosen ω determine the transition rates in leading order perturbation theory.

Problem 2: Selection Rules for One-Photon Absorption in Hydrogen Atoms

Determine the selection rules for one-photon absorption processes in the hydrogen atom, i.e., for which combination of quantum numbers n, ℓ, m for the initial state and n', ℓ', m' for the final state one can expect non-zero absorption rates. Express for this purpose the operator $\vec{r}$ in the transition dipole moment through spherical tensor operators and employ the Wigner-Eckart theorem.

Problem3: Ethylene π-Electron States

The molecule ethylene C_2H_4 has two $2p_z$ electrons on each of its carbons. Denoting the atomic orbitals (including spin) by $|j, \sigma\rangle$, $j = 1, 2$; $\sigma = \pm\frac{1}{2}$ assume that the Hamiltonian for the system is

$$H \;=\; -t \sum_\sigma \left(c^\dagger_{1\sigma} c_{2\sigma} \;+\; c^\dagger_{2\sigma} c_{1\sigma} \right) \;+\; U \left(c^\dagger_{1\alpha} c_{1\alpha}\, c^\dagger_{1\beta} c_{1\beta} \;+\; c^\dagger_{2\alpha} c_{2\alpha}\, c^\dagger_{2\beta} c_{2\beta} \right) \,. \tag{2}$$

Here α, β denote spin-up ($|\frac{1}{2}, \frac{1}{2}\rangle$) and spin-down ($|\frac{1}{2}, -\frac{1}{2}\rangle$) states, respectively.

(a) State all possible states of the system in the given basis of single electron states.

(b) Determine the Hamiltonian matrix in the basis of all 2-electron states.

(c) State all triplet states and their energies.

(d) Determine the energy of the lowest singlet state (ground state) in 2nd order (with respect to U) perturbation theory. Note: In second order perturbation theory holds for the energy of a state $|0\rangle$ (in the usual notation)

$$E_o \approx \text{zero order} + \text{first order contributions} + \sum_{n, n \neq 0} \frac{\langle 0|V|n\rangle\langle n|V|0\rangle}{\epsilon_0 - \epsilon_n} . \qquad (3)$$

All material, e.g., books, class notes, are allowed in the exam, except solutions (your own or those of others) of the homework sets of this class.