Problem Set 1
Physics 481 / Spring 2000
Professor Klaus Schulten

Problem 1: Isotropic oscillator in magnetic field

Derive the spectrum and stationary state wave functions for a charged, spinless particle moving in a time-independent, homogenous magnetic field $\vec{B}_0$ and potential $V = \frac{1}{2} m\omega^2 r^2$.

Problem 2: Wave Packet Moving in a Magnetic Field

The wave function $\psi(\vec{r}, t)$ of an electron moving in a time-independent, homogeneous magnetic field $B_o \hat{e}_3$ described by the vector potential

$$\vec{A} = \frac{1}{2} B_o \hat{e}_3 \times \vec{r} \qquad (1)$$

is governed by the Schrödinger equation

$$i\hbar\partial_t \psi(\vec{r}, t) = \left[-\frac{\hbar^2}{2m_e} \nabla^2 + \frac{1}{8} m_e \omega_L (x_1^2 + x_2^2) + \frac{1}{2} \omega_L \hat{J}_3 \right] \psi(\vec{r}, t) \qquad (2)$$

where $\omega_L = eB_o/m_e$ is the Larmor frequency and where $\hat{J}_3$ is the quantum mechanical angular momentum operator (see class notes).

(a) Assume for the wave function the form

$$\psi(\vec{r}, t) = e^{ik_3 x_3} \, e^{-\frac{i}{2\hbar}\omega_L t \hat{J}_3} \phi(x_1, x_2, t) \qquad (3)$$

and demonstrate that $\phi(x_1, x_2, t)$ obeys

$$i\hbar\partial_t \phi(x_1, x_2, t) = \left[-\frac{\hbar^2}{2m_e} (\partial_1^2 + \partial_2^2) + \frac{1}{8} m_e \omega_L (x_1^2 + x_2^2) + \frac{\hbar^2 k_3^2}{2m_e} \right] \phi(x_1, x_2, t) \qquad (4)$$

(b) Describe the classical motion of an electron in the field (1) for the initial position $\vec{r}(0) = (x_{1o}, 0, 0)$ and the initial momentum $(0, p_{2o}, p_{3o})$.

(c) Determine the wave function $\phi(x_1, x_2, t)$ for the initial state

$$\phi(x_1, x_2, 0) = N \exp[-\frac{1}{2}(y_1 - y_1 o)^2] \exp[-\frac{1}{2} y_2^2] \qquad (5)$$

where $y_j = \sqrt{m_e \omega_L / 2\hbar} x_j$, $j = 1, 2$. (Hint: use the solution of Problem 0).

(d) Plot $f(x_1, x_2) = |\phi(x_1, x_2, t)|^2$ for various times $t = \pi n/2\omega_L, n = 0, 1, 2, 3, 4, 5$ and $x_{1o} = 1$ using the `mathematica` command `Plot3D[ f(x1, x2), {x1, -2, 2}, {x2, -2, 2}]`.

(e) Using the identity

$$e^{-\frac{i}{\hbar}\alpha\hat{J}_3} f(x_1, x_2) = f(x_1 \cos\alpha + x_2 \sin\alpha, -x_1 \sin\alpha + x_2 \cos\alpha) \qquad (6)$$

plot the probability density $f(x_1, x_2) = |\psi(x_1, x_2, x_3, t)|^2$ for the same parameters as in (d).

(f) To which classical initial conditions does the probability density in (e) correspond?

Problem 3: Gauge Transformation of a Wave Function

Describe a pair of wave functions for an electron moving in a time-independent, homogeneous magnetic field described through a vector potential $\vec{A} = B_o x_1 \hat{e}_2$ or $\vec{A} = -B_o x_2 \hat{e}_1$ and show that the wave functions are related to each other through a phase factor $\exp[-ie\chi(\vec{r})/\hbar]$ where $\chi(\vec{r})$ accounts for the gauge transformation connecting the two vector potentials stated above. Choose a simple pair of wave functions of your choice, but note that the wave functions must correspond to the same physical situation.

The problem set needs to be handed in by Tuesday, February 1, 2000 into the mail box of Gheorghe-Sorin Paraoan in Loomis.