Appendix to Problem 4c

For a double delta function potential at $x = 0$ and $x = a$, the wavefunctions in the three regions are given by

$$\Psi_I = Ae^{ikx} + Be^{-ikx} \quad x < 0$$

$$\Psi_{II} = Ce^{ikx} + De^{-ikx} \quad 0 < x < a$$

$$\Psi_{III} = Ee^{ikx} \quad x > a$$

The boundary conditions at $X = 0$ and $X = a$ give

$$A + B = C + D \tag{1}$$

$$ik(C - D - A + B) = f(C + D) \tag{2}$$

$$Ce^{ika} + De^{-ika} = Ee^{ika} \tag{3}$$

$$ik[Ee^{ika} - Ce^{ika} + De^{-ika}] = fEe^{ika} \tag{4}$$

From eqn. 3 and eqn. 4 we get

$$E = \frac{2ik}{2ik - f}C \tag{5}$$

$$D = \frac{fe^{ika}}{2ik - f}C \tag{6}$$

Substituting D in terms of C in eqn. 1 & eqn. 2 we get

$$A + B = \alpha C \tag{7}$$

and

$$ik(\beta C - A + B) = \alpha C \tag{8}$$

where

$$\alpha = \frac{2ik - f + fe^{ika}}{2ik - f}$$

$$\beta = \frac{2ik - f - fe^{ika}}{2ik - f}$$

Above equations give

$$C = \frac{2ik}{2ik - \alpha f} A \tag{9}$$

and

$$B = \frac{2i\alpha k - 2ik + \alpha f}{2ik - \alpha f} A \tag{10}$$

Putting the value of α we get

$$B = -\frac{-f^2 + f^2 e^{2ika} + 2ikf + 2ikf e^{2ika}}{-f^2 + 4k^2 + +4ikf + f^2 e^{2ika}} A \tag{11}$$