Problem Set 1
Physics 480 / Fall 1999
Professor Klaus Schulten

Problem 1: Your E-Mail Address

Send your e-mail address to Pinaki Sengupta at pinaki@mephisto.physics.uiuc.edu.

Problem 2: Moving Wave Packet for Free Particle

A free one-dimensional particle is described by the propagator

$$\phi(x, t|x_0, t_0) \;=\; \left[\frac{m}{2\pi i\hbar(t\,-\,t_0)}\right]^{\frac{1}{2}} \exp\left[\frac{i}{\hbar}\frac{m}{2}\frac{(x\,-\,x_0)^2}{t\,-\,t_0}\right] \tag{1}$$

(a) Determine analytically the state $\psi(x, t)$ of the system at times $t > t_0$ for the initial state

$$\psi(x_0, t_0) \;=\; \left[\frac{1}{\pi\delta^2}\right]^{\frac{1}{4}} \exp\left[-\frac{x_0^2}{2\delta^2} \,+\, \frac{ip_0x_0}{\hbar}\right] \tag{2}$$

employing

$$\psi(x, t) \;=\; \int_{-\infty}^{\infty} dx_0\, \phi(x, t|x_0, t_0)\, \psi(x_0, t_0)/;. \tag{3}$$

For this purpose carry through the algebra as outlined in teh class notes.
(b) Plot $|\psi(x, t)|^2$ for some representative times $t_1, t_2, \ldots$assuming an electron, a proton, a uranium atom and $\delta = 1$Å. Choose your own p_0 values.

Problem 3: Particle in Homogeneous Force Field — Path Integral

Describe the propagation of the quantum state $\psi(x, t)$ corresponding to the 1-dimensional motion of a particle in the potential $V(x) = -fx$. Star again from the class notes and carry through every step of the calculation in complete detail.
(a) State the Lagrangian $L(x, \dot{x}, t)$ of the system and derive the classical equation of motion through the Euler Lagrange equation.
(b) Determine the classical path $x_{cl}(\tau)$ with endpoints $x(\tau = t_0) = x_0$ and $x(\tau = t) = x$.
(c) Show that the classical action integral for the path determined in (b) is

$$S[x_{cl}(\tau)] \;=\; -\frac{1}{12}\frac{f^2T^3}{2m} \,+\, \frac{fT}{2}(x_0 \,+\, x) \,+\, \frac{m}{2T}(x_0 \,-\, x)^2 \tag{4}$$

where $T = t - t_0$.

(d) Argue that the propagator $\phi(x,t|x_0,t_0)$ defined through the path integral

$$\phi(x,t|x_0,t_0) \;=\; \int_{x(t_0)=x_0}^{x(t)=x} d[x(\tau)] \, \exp\left\{\frac{i}{\hbar}\, S[x(\tau)]\right\} \tag{5}$$

can be written

$$\phi(x,t|x_0,t_0) \;=\; \exp\left\{\frac{i}{\hbar}\, S[x_{cl}(\tau)]\right\} \phi_{free}(0,t|0,t_0) \tag{6}$$

where $\phi_{free}(0,t|0,t_0)$ is the propagator for a free particle, i.e., for a Lagrangian $L_{free}(x,\dot{x}) = \frac{1}{2}m\dot{x}^2$, which had been determined to be $\sqrt{m/2\pi i\hbar T}$.
(e) Show, using

$$\psi(x,t) \;=\; \int_{-\infty}^{\infty} dx_0\, \phi(x,t|x_0,t_0)\psi(x_0,t_0)\;, \tag{7}$$

that $\psi(x,t)$ for the initial state $\psi(x,t_0) = \exp[-\frac{x^2}{2\delta^2} + ik_0 x]/\sqrt[4]{\pi\delta^2}$ is

$$\psi(x,t) \;=\; \frac{1}{\left[\pi\delta^2(1+\frac{\hbar^2T^2}{m^2\delta^4})\right]^{\frac{1}{4}}} \exp\left[-\frac{(x-\frac{\hbar k_0 T}{m}-\frac{fT^2}{2m})^2}{2\delta^2(1+\frac{\hbar^2T^2}{m^2\delta^4})}(1-\frac{i\hbar T}{m\delta^2}) + \frac{i}{\hbar}(\hbar k_0 + fT)x - \frac{i}{\hbar}\int_0^T d\tau\, \frac{(\hbar k_0 + f\tau)^2}{2m}\right] \tag{8}$$

(f) Sketch the resulting probability distribution as a funtion of time.

The problem set needs to be handed in by Tuesday, September 14. The web page of Physics 480 is at
http://www.ks.uiuc .edu/Services/Class/PHYS480/