Problem Set 4
Physics 386 / Spring 1997
Professor Klaus Schulten

Problem 1 : Quantum Mechanical and Classical Particles

(a) Electrons of kinetic energy 100 eV travel a distance of 1 km. If the size of the initial wave packet is given as 1 micron, what is the size at the end of the travel?

(b) An arrow of mass 200 g travels for 10 s with a velocity of 10 m/s. If the initial wave packet has a size of 1cm, what is the size of the packet at its desination?

Note: in this problem the size of a wave packet is taken to be the uncertainty Δx in the position of the wave packet, defined as $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$.

Problem 2: Expectation Value of Momentum

The wave function of a particle is

$$\phi(x) = \sqrt{\frac{2\hbar^3}{\pi \delta^3}}\, e^{ik_o x} \, \frac{1}{x^2 + \frac{\hbar^2}{\delta^2}} \, . \tag{1}$$

Show that the wave function is normalized.

(b) Determine the momentum representation of the wave function.

(c) Show that the wave function is normalized in the momentum representation.

(d) Evaluate the momentum expectation value in the momentum representation.

Note:

$$\int_0^\infty dx \, \frac{\cos(bx)}{x^2 + a^2} = \frac{\pi}{2a}\, e^{-ba} \, . \tag{2}$$

Problem 3: Wave Packet

Derive the analytical expression for the wave packet, as stated in class, i.e., evaluate

$$\psi(x,t) \;=\; \frac{1}{\sqrt{2\pi\Delta k}\pi^{1/4}} \int_{-\infty}^{+\infty} dk \exp\left[-\frac{(k-k_o)^2}{2(\Delta k)^2} \;+\; ik\left(x - \frac{\hbar k}{2m}t\right)\right] \quad (3)$$

through completion of the square in the exponent and Gaussian integration.

This problem set needs to be handed in by Wednesday, March, 1997.