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A Regularized Approach to Color Constancy

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Abstract. Color constancy is the ability of color perception independent of the spectrum of the ambient illumination. We present an algorithm that makes use of biologically plausible assumptions concerning the spectra of illumination and surface reflectance in order to solve the undetermined problem of color constancy. We test the proposed algorithm by means of computer simulations and examine its range of performance.

1 Introduction

While looking at a colorful painting under morning light, evening light or a light bulb, we still perceive reds as reds and blues as blues, although the light falling on the photoreceptors of our eyes varies substantially with regard to its spectral composition under the different sources of illumination. Our visual system – and that of many other biological species – is able to discount the ambient light, without having any precise information about the composition of this light (see e.g. Kandel and Schwartz 1985; Livingstone and Hubel 1984; Ingle 1985; Werner et al. 1988). The ability of color perception largely independent of the ambient light is called color constancy.

In this paper we present an algorithm that computes the spectral surface reflectance, i.e. a measure for a color sensation independent of the ambient light. As the information available to the first stages of biological vision is not sufficient to calculate the surface reflectance, additional constraints must be imposed for solving the problem of color constancy. The necessary constraints are based on biologically plausible assumptions and are formulated in the framework of standard regularisation theory (Poggio et al. 1985). These constraints could then correspond to the minimal biological conditions under which color constancy can be realized. The suggested algorithm could serve in an artificial visual system for color perception.

In Sect. 2 we will give some preliminary definitions and then state in Sect. 3 the problem of color constancy in a mathematical form. Section 4 will contain some remarks about existing algorithms yielding color constancy. We will then present in Sect. 5 our approach and discuss the performance of the proposed algorithm in Sect. 6.

2 Definitions

Consider a two-dimensional discretized scene with coordinates (x, y), $x=1,2,\ldots,N_x$, $y=1,2,\ldots,N_y$. The total number of pixels is $N=N_xN_y$. The scene is illuminated by a source of light. Each point (x, y) of the scene reflects a portion of the incoming light $E(x, y, \lambda)$, where the proportionality coefficient $R(x, y, \lambda)$ is the spectral surface reflectance. For simplicity we assume that the reflection is specular and that the angle between the light source and the scene matches the angle between the observer and the scene. The spectrum reflected from the scene $E(x, y, \lambda)R(x, y, \lambda)$ falls on a planar grid, where each node (x', y') corresponds to the three types of color receptors, namely the cones. As each cone only obtains information from one point of the scene, the coordinates of the array of receptors (x', y') can be identified with the coordinates of the scene (x, y).

Each color receptor, labeled by j, is specified by its spectral sensitivity $S_j(\lambda)$. The information that the color receptor j at location (x, y) transmits to the artificial visual system is

$$\varrho_j(x, y) = \int d\lambda S_j(\lambda) E(x, y, \lambda) R(x, y, \lambda), \qquad (1)$$

where the integral is taken over the visible range of the spectrum. $\varrho_j(x, y)$ is the only information available to the abstract visual system. Therefore, we are confronted with the problem of determining from $3N$ knowns, i.e. $\varrho_j(x, y)$, the spectra $E(x, y, \lambda)$ and $R(x, y, \lambda)$ that are functions of an infinite number of degrees of freedom. In order to solve this problem one first needs to reduce the infinite space of possible solutions.

3 Models for Finite Bases

A large class of illumination spectra can be described by the spectra of ideal Planckian radiators (Wyszecki and Stiles 1982). Judd et al. (1964) have examined daylight distributions and found that these spectra can be described by three basis functions. Maloney (1985) conducted a data analysis for Planckian radiators with correlated color temperature between 1900 K and 10300 K. He found that three functions could account for all the variance of the data within numerical precision. Following this author, we represent the distribution of ambient light by a linear, three-dimensional model, i.e. as

$$E(x, y, \lambda) = \sum_{i=1}^{3} \varepsilon_i(x, y) E_i(\lambda), \tag{2}$$

with fixed known basis functions $E_i(\lambda)$.

A similar reduction of the number of degrees of freedom is possible for the surface spectral reflectance. Stiles et al. (1977) showed that many surface reflectance spectra are smooth and low-frequency functions of wavelength and, thus, can be described by a finite number of basis functions. Similarly, Maloney (1985) argued that, because of molecular processes, reflectance spectra should be low-frequency functions. Cohen (1964) analysed a set of randomly chosen Munsell samples and found that three functions can account for 0.992 of the variance of the data. Buchsbaum and Gottschalk (1984) computed the chromaticity coordinates of frequency limited functions and proposed that the space of color signals may be three-dimensional. We will assume in the following that the surface spectral reflectance $R(x, y, \lambda)$ can be described by three basis functions $R_l(\lambda)$ and $3N$ coefficients $\beta_l(x, y)$:

$$R(x, y, \lambda) = \sum_{l=1}^{3} \beta_l(x, y) R_l(\lambda). \tag{3}$$

Combining (1), (2), and (3) yields

$$\varrho_j(x, y) = \sum_{i,l=1}^{3} A^j_{il} \varepsilon_i(x, y) \beta_l(x, y), \tag{4}$$

where the tensor A is given by

$$A^j_{il} = \int d\lambda S_j(\lambda) E_i(\lambda) R_l(\lambda). \tag{5}$$

From now on, we assume that the illumination is either constant, i.e. $\varepsilon_i(x, y) = \varepsilon_i$, or else displays a linear gradient over the scene, i.e. $\varepsilon_i(x, y) = \varepsilon_i + \gamma_i x + \delta_i y$. Solving the problem of color constancy is then reduced to computing from the $3N$ signals $\varrho_j(x, y)$ given by (4) three coefficients ε_i (in the case of constant illumination) or nine coefficients ε_i, γ_i, and δ_i (in the case of a linear illumination gradient) as well as $3N$ coefficients $\beta_l(x, y)$.

4 Previous Models

Land's retinex algorithm (Land 1983) certainly is the best-known algorithm realizing color constancy. Land calculates separately for each color channel the surface reflectance (called lightness value) at some point of a scene by measuring the reflectance along randomly chosen paths over the scene passing through this point. Brainard and Wandell (1986) analysed the retinex algorithm and compared the results of the algorithm with those of psychophysical experiments. They found that the computed lightness values depend too much on the composition of the scene and that an optimal choice of the path length is problematic. In a newer version of the retinex algorithm (Land 1986) the lightness value at a given point is no longer calculated by means of one-dimensional paths, but by a two-dimensional normalisation procedure in the neighborhood of this point. To our knowledge, precise and quantitative results concerning color constancy have not been published in that case.

Several approaches exist which associate color constancy with adaption mechanisms (see e.g. West and Brill 1982 or Hunt 1987). However, such association does not appear convincing in view of the fact that adaptation is a temporal effect, whereas color constancy is realized almost instantaneously (Land 1983).

Hurlbert and Poggio (1988) developed a color algorithm from a set of examples which as inputs comprised the signals $\varrho_j(x, y)$ and as outputs (solutions) the reflectance coefficients $\beta_l(x, y)$. They, as well as other authors (for a review, see Hurlbert 1986), require in their approach equality of the basis functions $E_i(\lambda)$ and $R_i(\lambda)$ and assume that these functions are orthogonal to the spectral sensitivities $S_i(\lambda)$. These assumptions are not met by the sensitivity curves of human cones and by the basis functions for illumination and reflectance spectra obtained by data analyses (Stiles et al. 1977; Wyszecki and Stiles 1982; Judd et al. 1964).

By imposing the condition that the number of color receptors is larger than the number of degrees of freedom of the reflectance function $R(x, y, \lambda)$, and that a sufficient number of different surface reflectance spectra are present in the scene, the problem of color constancy as stated in (4) can be solved (Maloney and Wandell 1986; Yuille 1987). The first assumption implies that one either deals with more than three color receptors or that only reflectance and illumination spectra described by two basis functions [cf. (3)] can be recovered. However, examination of the frequency limits of reflectance spectra implies that at least three basis functions are needed for a proper description of these spectra. Therefore, more than three color receptors would be needed for computation of the spectra. The implication for biological vision would be a contribution of the rods to color vision. However, no rods are present in the central part of the fovea and a possible contribution of rods for color vision has not been demonstrated (Wyszecki and Stiles 1982).

A different approach consists in reducing the infinite space of possible solutions to (4) by further, biologically plausible assumptions concerning the solutions. This approach was proposed by Poggio and coworkers (1985) for solving different low level vision problems and we will follow it in the context of color constancy.

5 A Regularized Approach to Color Constancy

Many problems in early vision (e.g. edge detection, stereo vision, color vision) are underdetermined in the sense that the number of unknowns exceeds the number of knowns. The information about the scene available to the photoreceptors does therefore not allow a unique interpretation of the scene. Such a problem, where a unique solution that depends continuously on the data does not exist, is called ill-posed (Tikhonov and Arsenin 1977). In order to solve an ill-posed equation $Hz=y$, one chooses stabilizing functionals P_n, such that the minimum of the cost function

$$\|Hz-y\|^2+\sum_n \lambda_n \|P_n z\|^2 \tag{6}$$

yields a unique solution z. A sufficient condition for the uniqueness of the solution is the linearity of H and P_n. $\|\cdot\|$ is an appropriately chosen norm and the regularizing parameters λ_n control the amplitudes of the stabilizing terms.

Minimizing (6) in the context of color constancy is equivalent to solving (4) under additional constraints. These constraints express assumptions that are possibly made during the visual process of color perception. The first assumption about the scene concerns the spectrum of the light source. The reddish or bluish look of all objects in a discotheque, where tinted light bulbs are used, is a well-known example for the breakdown of color constancy. We therefore require the spectrum of ambient light to be as "white" as possible, i.e. as close as possible to the spectrum of average daylight D_{65} (Wyszecki and Stiles 1982). In the case of constant illumination the corresponding stabilizing functional will take the form

$$P_1=\sum_i (\varepsilon_i-\varepsilon_i(D_{65}))^2\,, \tag{7}$$

where $\varepsilon_i(D_{65})$ are the coefficients of the spectrum D_{65} in the basis used.

The assumption expressed in (7) alone does not lead to a unique solution. If ε_i and β_l minimize

$$\sum_j \sum_{x,y} \Big(\sum_{il} A^j_{il}\varepsilon_i(x,y)\beta_l(x,y)-\varrho_j(x,y)\Big)^2+\lambda_1 P_1\,,$$

then ε_i/a and $a\cdot\beta_l$ do so as well. Therefore, we require the averages of the reflectance coefficients in each color channel to take a fixed value b_l, corresponding to an average color sensation of grey. This situation is matched in case of a scene with many different color patches, but also in case of a single color stimulus, surrounded by a large enough grey area. Psychophysical experiments have yielded, in fact, color constancy in these two cases (Arend and Reeves 1986). The assumption of fixed averages can be forced by the stabilizing functional

$$P_2=\sum_l \left(\sum_{x,y}\frac{\beta_l(x,y)}{N}-b_l\right)^2. \tag{8}$$

In case of linear gradients the spectrum of ambient light can be described by the coefficients $\varepsilon_i(x,y)=\varepsilon_i+\gamma_i x+\delta_i y$. We require the gradients to be small and express this with the additional term

$$P_3=\sum_i \gamma_i^2+\delta_i^2\,. \tag{9}$$

In order to calculate the spectra of illumination and reflectance, we need to minimize the cost function

$$E=\sum_j \sum_{x,y}\Big(\sum_{il} A^j_{il}\varepsilon_i(x,y)\beta_l(x,y)-\varrho_j(x,y)\Big)^2+\sum_{n=1}^{3}\lambda_n P_n\,. \tag{10}$$

E is locally convex in each variable and, therefore, globally convex (Tikhonov and Arsenin 1977). We can use a steepest descent procedure, updating the variables if the corresponding change lowers the value of E. The configuration (ε_i, β_l) at the global minimum corresponds then to the optimal visual interpretation of the scene. Since we consider only non-fluorescent objects, we allow the coefficients $\beta_l(x,y)$ to take discrete values, such that the the reflectance spectrum is bounded between 0 and 1. The resolution of $\beta_l(x,y)$ is limited to ± 0.01, which is justified by the fact that the number of discriminable colors is limited (Richter 1981). Furthermore, too high of a resolution slows down the convergence of the minimization procedure. The criteria leading to a choice of regularizing parameters λ_n at the start of the minimization procedure will be discussed in Sect. 6.

Instead of starting with random variables, starting with a configuration that solves (4) and therefore corresponds to a minimum of E in a restricted space, will reduce the computational effort. An important reduction of computational effort during the minimization procedure can be achieved by realizing that the reflectance spectra are either constant or show discontinuities. Since the illumination is supposed to be smooth, jumps in the color signals $\varrho_j(x, y)$ can be related to locations where the reflectance coefficients are discontinuous. Therefore, we first detect discontinuities in the color signals $\varrho_j(x, y)$ by means of the Laplace operator (Marr 1982). The values of $\beta_l(x, y)$ are then only computed at the discontinuities and the missing $\beta_l(x, y)$ are interpolated within surfaces of constant reflectance.

In case of constant illumination the algorithm can be reduced to a fully deterministic procedure. Averaging (4) over the scene and interpreting the averages $\langle\beta_l\rangle$ as free parameters α_l set by the assumptions made about the scene, yields the illumination coefficients

$$\varepsilon_i = \sum_k (A_+^{-1})_{ik} \langle\varrho_k\rangle, \tag{11}$$

where the matrix A_+^{-1} is the inverse of the matrix A_+ with coefficients

$$(A_+)_{mn} = \sum_k \alpha_k A_{nk}^m. \tag{12}$$

Combining (4) and (11) leads to the expression for the reflectance coefficients

$$\beta_l(x, y) = \sum_k (B^{-1})_{lk} \varrho_k(x, y), \tag{13}$$

where the matrix B is given by

$$B_{mn} = \sum_i A_{in}^m \sum_k (A_+^{-1})_{ik} \langle\varrho_k\rangle. \tag{14}$$

The parameters α_l are determined as to minimize the reduced cost function $E(\boldsymbol{\alpha})$

$$E(\boldsymbol{\alpha}) = \lambda_1 \sum_i \left(\sum_k \langle\varrho_k\rangle ((A_+^{-1})_{ik} - \varepsilon_i(D_{65})\right)^2 + \lambda_2 \sum_l (\alpha_l - b_l)^2. \tag{15}$$

As the minimization of this expression is analytically intractable, it has to be solved by a standard numerical minimization procedure (e.g. Powell's method, cf. Press et al. 1986). Since the reduced cost function does not depend on all variables, but only on the parameters α_l, the computational effort is considerably reduced in case of the deterministic procedure.

A deterministic version of the algorithm can also be used in case of gradients in the ambient light, if the tensor A_{il}^j is diagonal ($A_{il}^j \sim \delta_{il}\delta_{ij}$). But, as stated above, this is a somewhat artificial situation that does not apply to realistic basis functions and sensitivity curves of human or animal cones.

We want to stress here that, just as it is the case for the human visual system, the algorithm cannot in general yield perfect color constancy. The performance of the algorithm can be seen as a balance between two extreme situations: Setting the parameter λ_1 equal to zero would lead to perfect color constancy which is in general not possible unless all scenes do have a grey reflectance average. On the other hand, setting λ_2 equal to zero would correspond to the "naive" view of color perception, namely that the illumination is always white.

6 Results of Simulations

We tested the performance of the proposed algorithm by computing the reflectance spectrum of a so-called Mondrian, the prototype of the two-dimensional scene (Land 1983). Our Mondrian consists of ten rectangulary shaped, homogeneously colored regions of varying size. The aim of the algorithm is to determine the reflectance coefficients β^{out} of the Mondrian regions. We have chosen two displays of the resulting β^{out}. In the first kind of display the coefficients are plotted as a function of the pixel location (x, y) for each color channel separately. The second kind of display is more intuitive and presents the reflectance coefficients as colors on a monitor. This display allows a qualitative judgement about the performance of the algorithm. As the reflectance coefficients do not represent an absolute measure for color perception and, furthermore, depend on the chosen basis we actually display the color signals ϱ^{ref} and ϱ^{out}, where $\varrho_j^{ref} = \sum_{il} A_{il}^j \varepsilon_i(D_{65}) \beta_l^{in}(x, y)$ and $\varrho_j^{out} = \sum_{il} A_{il}^j \varepsilon_i(D_{65}) \beta_l^{out}(x, y)$ rather then the input coefficients β^{in} and the calculated coefficients β^{out}. ϱ^{ref} and ϱ^{out} correspond to the appearance of the input Mondrian and the computed Mondrian under standard daylight D_{65}. Thus, a comparative judgement of the given and the computed reflectance spectra is possible.

As a color display we used an *Apple RGB Monitor* with a resolution of 480×640 pixels. As this monitor can display only 256 colors simultaneously, we limited ourselves to Mondrians of 10×10 pixels, where each pixel is enlarged to a square of 30×30 pixels on the screen. The color signals were displayed by linearly assigning RGB-values of the monitor to ϱ_j, such that the extremes of the possible reflectance spectra – namely $R(x, y, \lambda)$ equals 0 or 1 – correspond to black or white. Considering two surfaces A and B with reflectance coefficients β^A and β^B and corresponding color signals ϱ^A and ϱ^B, we can ask what the minimal difference per color channel $\Delta\varrho_j$ (where $\Delta\varrho_j = |\varrho_j^A - \varrho_j^B|$), must be, such that the colors of A and B are perceived as different. From data about color discrimination (cf. e.g. Richter 1981) it follows that, for adjacent surfaces, $\Delta \varrho_j$ should be between 0.01 and 0.02. By informal experiments we came to the same conclusions. But this result constitutes only an approximate guideline, since the discriminability of colors is not homogeneous in color space and, to a large extent, depends on the setup of the experiments (Wyszecki and Stiles 1982). Furthermore, in realistic stiuations, as opposed to laboratory conditions, the difference $\Delta \varrho_j$ will have to be much larger in order to yield a significant difference in color sensation.

In our simulations we used the spectral response functions of the cones derived by Judd (Wyszecki and Stiles 1982). As basis functions for the reflectance spectrum we chose Fourier functions as suggested by Wandell (1987). For the spectra of ambient light we used the three distributions obtained by a principal component analysis of a set of daylight distributions (Wyszecki and Stiles 1982). However, in the case of the color displays, we used Mondrians with somewhat artificial reflectance spectra described by three Gaussians peaked at different wavelengths. The reason for this choice is that, when looking at the screen, the displayed color signals ϱ are distorted by a second convolution with the sensitivity curves of the human cones. We chose Mondrians with very peaked reflectance spectra, because they allow a clearer and more vivid display. We want to stress, however, that this choice of basis functions in the case of the Mondrians used for color display does not affect in any way the results of our simulations.

With the exception of the case where a gradient in illumination was presented we used the deterministic version of the algorithm for all simulations. However, in order to show the convergence of the cost function and the time evolution of the reflectance coefficients during the stochastic procedure, we present results of the stochastic algorithm. As far as the optimal choice of regularization parameters λ_n is concerned, there exist a number of procedures for evaluating or adjusting the values of λ_n during the minimization of the cost function (Tikhonov and Arsenin 1977). We found that in practice it is sufficient to choose the parameters λ_n such that the amplitudes of the corresponding stabilizing terms P_n are of the same order of magnitude as the main term of the cost function. However, if we have an a priori knowledge about the composition of the scene or the ambient light, we can set the parameters in a way that will take this knowledge into account. In all the presented simulations the parameters λ_n were set to the same fixed values ($\lambda_1 = 10$, $\lambda_2 = 10^6$, and $\lambda_3 = 10^3$ in the case of the stochastic algorithm, $\lambda_1/\lambda_2 = 10^{-2}$ in the case of the deterministic algorithm).

Figure 1 shows the logarithmic convergence of the different terms of the cost function in the case of *standard* conditions, i.e. average daylight D_{65} and grey average of the reflectances. The top left graph displays the total cost function E, whereas the main term, corresponding to (4) is shown in the top right graph. The stabilizing terms P_1, forcing the illumination to be close to average daylight and P_2, forcing the average of the reflectance spectrum to correspond to grey are shown in the bottom left and right graphs. One can see that all the terms decay on the same time scale to their asymptotic values. Figure 2 illustrates the performance of the algorithm for the same case as Fig. 1. The

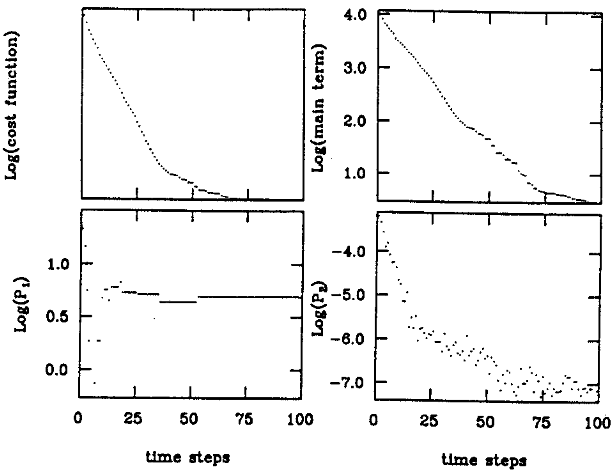

Fig. 1. Logarithmic convergence of the different terms of the cost function as a function of time steps in the case of standard conditions. Top left: total cost function E, top right: main term, bottom left: illumination term P_1, bottom right: grey average term P_2

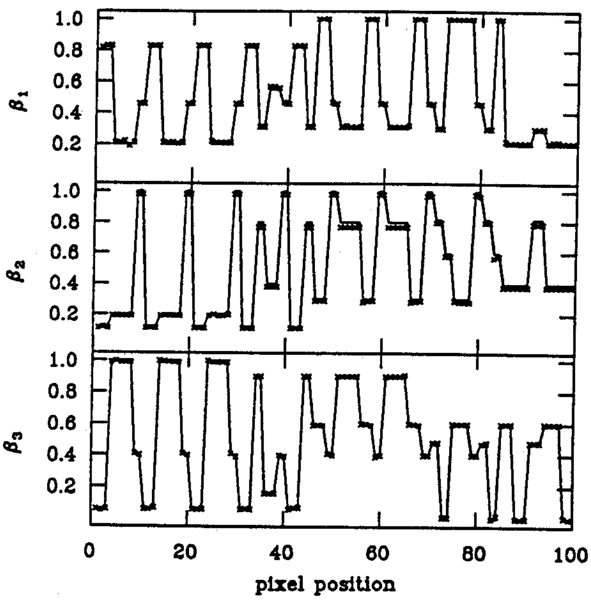

Fig. 2. Computed reflectance coefficients β^{out} (crosses) versus input coefficients β^{in} (solid line) as a function of pixel position in the case of the standard Mondrian

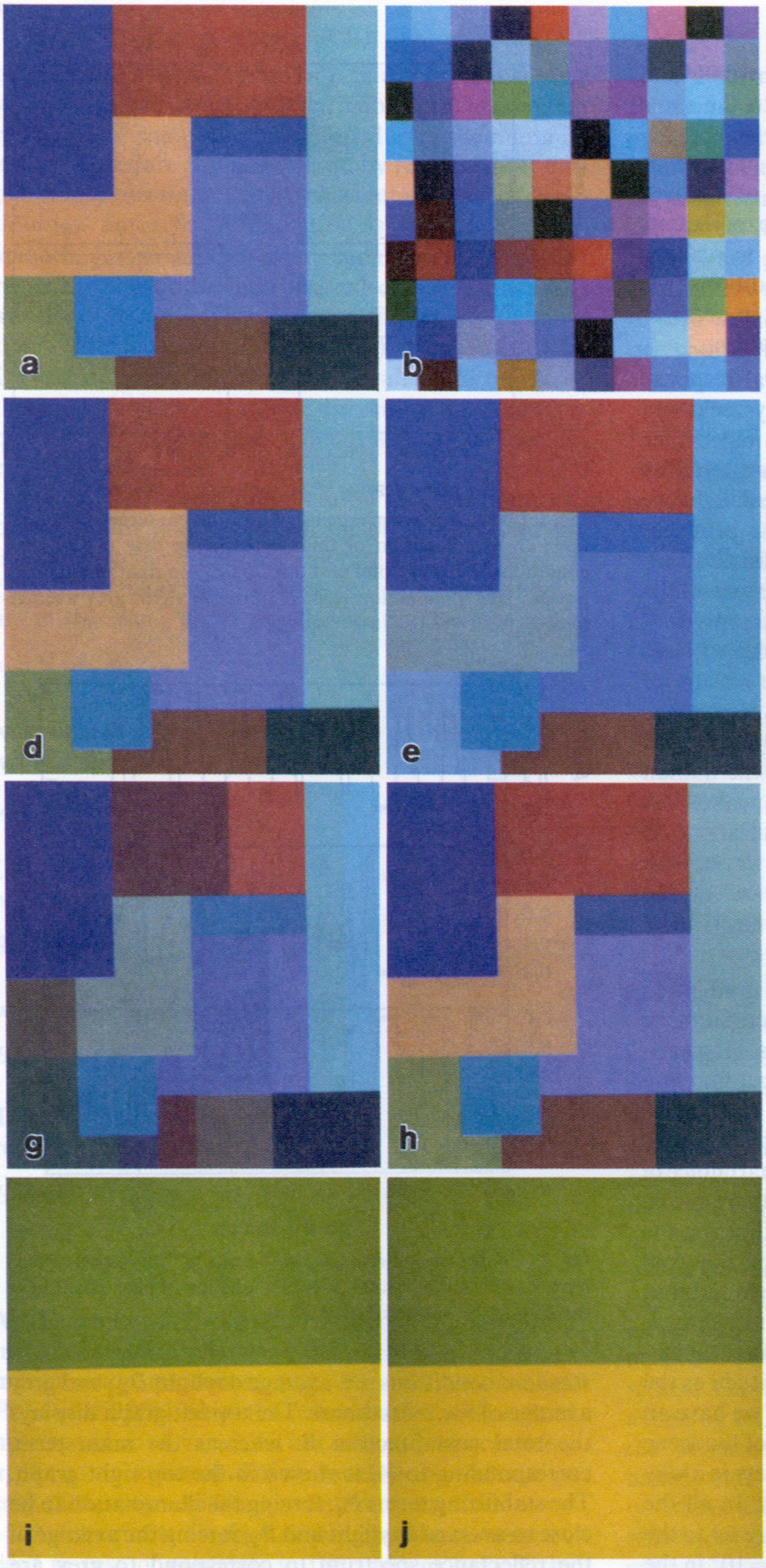

Fig. 3a–k. See text for definitions of ϱ^{in}, ϱ^{out}, and ϱ^{ref}. **a** ϱ^{ref} for a Mondrian with grey reflectance average (standard Mondrian). **b** ϱ^{out} for the standard Mondrian under average daylight D_{65} at the start of the simulation. **c** same as **b** after 30 steps. **d** same as **b** after convergence of the algorithm. **e** input to the algorithm ϱ^{in} in the case of extreme daylight with correlated color temperature 10000 K. **f** output ϱ^{out} corresponding to the input displayed in **e**. **g** input to the algorithm ϱ^{in} for a Mondrian with a linear gradient of illumination. **h** output ϱ^{out} corresponding to the input displayed in **g**. **i** ϱ^{ref} of a two-color stimulus with non-grey reflectance average. **j** ϱ^{out} corresponding to **i**, the illumination was average daylight D_{65}. **k** ϱ^{out} corresponding to **i**, the illumination was 10000 K-daylight

computed reflectance coefficients β^{out}, displayed by crosses, match well the input coefficients β^{in} represented by solid lines. The colored Fig. 3a and d displaying the corresponding ϱ^{ref} and ϱ^{out} shows what the input and the computed Mondrians would look like under standard daylight. Figure 3b and c displays ϱ^{out} for the standard Mondrian at the start of the simulation and after 30 time steps. While the pixels have totally random reflectance values at the start of the simulation, a considerable degree of convergence has been attained after 30 time steps.

We have examined the performance of the algorithm when the illumination is changed. Figure 3e shows the input to the algorithm $\varrho^{in} = \sum_{il} A^j_{il} \varepsilon^{in}_i \beta^{in}_l$ under a daylight illuminant with 10000 K correlated color temperature, i.e. an extreme of daylight (Wyszecki and Stiles 1982). Figure 3f displays the corresponding ϱ^{out}, i.e. the computed Mondrian under standard daylight. Comparison of Fig. 3a and f shows that the computed reflectance spectrum recovers well the input spectrum. Figure 4 shows a quantitative estimate of the averaged discrepancy between computed and given color signals $\langle \Delta\varrho \rangle$ under daylights with correlated color temperatures between 4000 K and 10000 K. The abscissa Δ_ε is a measure for the discrepancy between the actual spectrum of illumination and the standard daylight spectrum. We define Δ_ε and $\langle \Delta\varrho \rangle$ by

$$\langle \Delta\varrho \rangle = \sum_j \sum_{x,y} |\varrho^{out}_j(x, y) - \varrho^{ref}_j(x, y)|/3N$$

and $\Delta_\varepsilon = \sum_i |\varepsilon^{in}_i - \varepsilon_i(D_{65})|/3$. The blue and red extremes of daylight with correlated color temperatures of 10000 K and 4000 K correspond to values Δ_ε of 0.54 and 1.58. Figure 4 shows that, for most phases of daylight, the error $\langle \Delta\varrho \rangle$ is close to the threshold of discriminability mentioned above.

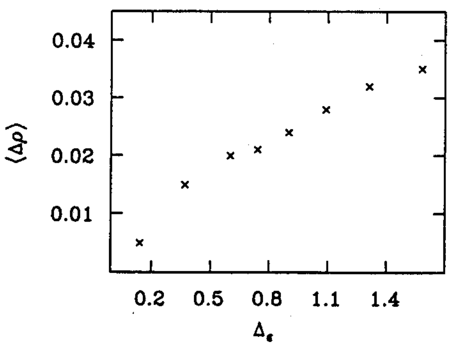

Fig. 4. Averaged deviation of the computed color signals from the reference color signals $\langle \Delta\varrho \rangle$ as a function of the averaged discrepancy between the actual spectrum of illumination and the standard daylight spectrum Δ_ε. See text for definitions of $\langle \Delta\varrho \rangle$ and Δ_ε

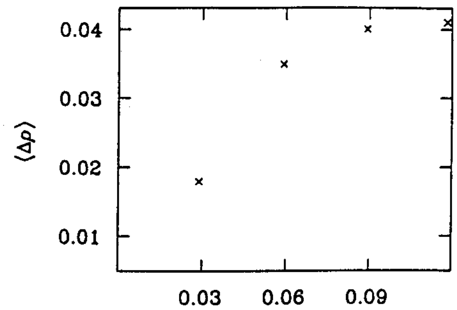

Fig. 5. Averaged deviation of the computed color signals from the reference color signals $\langle \Delta\varrho \rangle$ as a function of the deviation from the grey average condition $\Delta_{\langle\beta\rangle}$. See text for definitions of $\langle \Delta\varrho \rangle$ and $\Delta_{\langle\beta\rangle}$

Figure 3g shows the input ϱ^{in} when the Mondrian is illuminated with a linear horizontal gradient. The corresponding output ϱ^{out} is displayed in Fig. 3h. Comparison of Fig. 3h and a shows that the computed reflectance spectrum agrees well with the input spectrum.

Similarly, we examined the performance of the algorithm when the average of the reflectance spectrum over the scene does not correspond to grey. We quantify the deviation from the grey average condition by $\Delta_{\langle\beta\rangle} = \sum_l |\langle \beta^{in}_l \rangle - b_l|/3$, where b_l are the reflectance coefficients of a spectrum yielding a color sensation of grey. Figure 5 shows the average discrepancy between output and reference color signals $\langle \Delta\varrho \rangle$ as a function of $\Delta_{\langle\beta\rangle}$ in the case of standard daylight. Even for a Mondrian composed of mostly blue patches (where $\Delta_{\langle\beta\rangle}$ equals 0.12), $\langle \Delta\varrho \rangle$ is small.

Presenting as input this same Mondrian under illuminations differing from the standard daylight will yield correct interpretations of the reflectance spectrum for daylights with correlated color temperatures in the range of approximately 5500 K to 8000 K (i.e. $\Delta_\varepsilon \leqq 0.30$). However, when neither the condition of white illumination nor the condition of grey reflectance are approximately fulfilled, the algorithm will not give a correct interpretation of a scene. Figure 3i–k illustrates this fact. Figure 3j and k shows the computed color signals ϱ^{out} of a two-color stimulus with non-grey average under standard daylight and under 10000 K-daylight. They should be compared with the corresponding ϱ^{ref} in Fig. 3i. Whereas the reflectance spectrum has been correctly recovered under average daylight, one can see that in the case of 10000 K-daylight the computed reflectance coefficients β^{out} do not match the input coefficients β^{in}.

7 Conclusions

This paper presents an algorithm for computing the surface reflectance of a two-dimensional scene, independently of the ambient illumination. We tested the algorithm for different Mondrians under varying phases of daylights and linear gradients in the illuminations. In most of the situations tested, the reflectance spectrum was well recovered. We gave a quantitative estimate of the range of performance of the described algorithm.

In order to recover the reflectance spectrum we are confronted with an ill-posed problem that can be solved by standard regularization methods. Poggio and Koch (1985) showed that problems of this type can be implemented by analogue networks. Therefore, a neural network model could be used as well for realizing the proposed algorithm.

It would be interesting to compare the results of our simulations with data about color constancy (see e.g. Arend and Reeves 1986). We could then adjust the free parameter of the algorithm – namely the regularizing parameters λ_n – such that the results of our simulations match the data from experiments about human color constancy.

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