NAMD Tutorial (Part 2)

2 Analysis

2.1 Equilibrium

2.1.1 RMSD for individual residues
2.1.2 Maxwell-Boltzmann Distribution
2.1.3 Energies
2.1.4 Temperature distribution
2.1.5 Specific Heat

2.2 Non-equilibrium properties of protein

2.2.1 Heat Diffusion

2.2.2 Temperature echoes
Temperature Echoes in Proteins

- Coherent motion in proteins: Echoes
- Generation of echoes in *ubiquitin* via velocity reassignments

1) Temperature quench echoes
2) Constant velocity reassignment echoes
3) Velocity reassignment echoes

\[ T(t) = \frac{2}{(3N - 6)k_B} \sum_{n=1}^{3N-6} \frac{m_n v_n^2(t)}{2} \]
Temperature Echoes

- are sharp, resonance-like features in the time evolution of the protein's temperature
- can be produced through 2 consecutive velocity reassignments

protein in equilibrium

synchronization signal

probing signal

temperature echoes (a kind of "interference" effect!)

Temperature

Time
Velocity Reassignments

- protein \( \approx \) collection of weakly interacting harmonic oscillators having different frequencies
- at \( t_1 = 0 \) the 1st velocity reassignment: \( v_i(0) = \lambda_1 u_i \) synchronizes the oscillators (i.e., make them oscillate in phase)
- at \( t_2 = \tau \) (delay time) the 2nd velocity reassignment: \( v_i(\tau) = \lambda_2 u_i \) probes the degree of coherence of the system at that moment
- degree of coherence is characterized by:
  - the time(s) of the echo(es)
  - the depth of the echo(es)

\( \lambda_1 = \lambda_2 = 0 \Rightarrow \) temperature quench
\( \lambda_1 = \lambda_2 = 1 \Rightarrow \) constant velocity reassignment
\( \lambda_1 \neq \lambda_2 \neq 1 \Rightarrow \) velocity reassignment
Producing Temperature Echoes by Velocity Reassignments in Proteins

- Equilibration: coupling to a heat bath; $T = T_0$
- Equilibrium: no heat bath coupling
- Relaxation: no heat bath coupling
- Relaxation: no heat bath coupling

Temperature quench echoes: $v_i(0) = v_i(\tau) = 0$

Const velocity reassignment echoes: $v_i(0) = v_i(\tau) = u_i$
Generating T-Quench Echo: Step 1

- your system is ubiquitin (1UBQ) in vacuum, pre-equilibrated at $T_0=300K$
- run all simulations in the microcanonical (NVE) ensemble
- psf, pdb and starting binary coordinate and velocity files are available in "common/"
- use NAMD2 configuration file "equil.conf" located in "01_equil_NVE/" to complete a 500 fs (# simulation steps) run
- extract the temperature time series $T(t)$ from the NAMD2 log (output) file
- plot $T(t)$
- calculate: $\langle T \rangle$, $\sqrt{\langle T^2 \rangle}$, $C_T = \langle \delta T(t) \delta T(0) \rangle$
Temperature Autocorrelation Function

\[ \Delta T(t) = T(t) - \langle T(t) \rangle \]

\[ C(t) = \langle \Delta T(t) \Delta T(0) \rangle \]

\[ \rightarrow C(t_i) \approx \frac{1}{N-i} \sum_{n=1}^{N-i} \Delta T(t_{n+i}) \Delta T(t_n) \]

\[ C(t) = C(0) \exp \left( -\frac{t}{\tau_0} \right) \]

Temperature relaxation time:

\[ \tau_0 \approx 2.2 \text{ fs} \]

Mean temperature:

\[ \langle T \rangle = 299 \text{ K} \]

RMS temperature:

\[ \sqrt{\langle \Delta T^2 \rangle} = \sqrt{C(0)} = 6 \text{ K} \]
Generating T-Quench Echo: Step 2

Perform the 1st temperature quench
- start a new simulation using configuration file "quench.conf" located in "02_quencha/"
- use the restart coordinate file from the previous run
- set all velocities to zero (i.e., set T=0)
- run the simulation for $\tau$ number of steps (time step = 1 fs)
- extract (from the log file) and plot $T(t)$
Generating T-Quench Echo: Step 3

Perform the 2\textsuperscript{nd} temperature quench

- start a new simulation using configuration file "quench.conf" located in "03_quenchb/"
- use the restart coordinate file from the previous run
- set again all velocities to zero (i.e., set $T=0$)
- run the simulation for $3\tau$ number of steps (time step = 1 fs)
- extract (from the log file) and plot $T(t)$

You should discover a temperature echo at $2\tau$
Explanation of the T-Quench Echo

**Assumption**: protein \( \approx \) collection of weakly interacting harmonic oscillators with dispersion \( \omega = \omega_\alpha, \alpha = 1, \ldots, 3N - 6 \)

**Step 1**: \( t < 0 \)
\[
x(t) = A_0 \cos(\omega t + \theta_0)
\]
\[
v(t) = -\omega A_0 \sin(\omega t + \theta_0)
\]

**Step 2**: \( 0 < t < \tau \)
\[
x_1(t) = A_1 \cos(\omega t + \theta_1)
\]
\[
v_1(t) = -\omega A_1 \sin(\omega t + \theta_1)
\]
\[
\begin{align*}
A_1 &= A_0 \cos \theta_0 \\
\theta_1 &= 0
\end{align*}
\]

**Step 3**: \( t > \tau \)
\[
x_2(t) = A_2 \cos(\omega t + \theta_2)
\]
\[
v_2(t) = -\omega A_2 \sin(\omega t + \theta_2)
\]
\[
\begin{align*}
A_2 &= A_1 \cos \omega \tau \\
\theta_2 &= -\omega \tau
\end{align*}
\]
T-Quench Echo: Harmonic Approximation

\[ T(t) \approx \frac{T_0}{4}\left[1 - \langle \cos(2\omega(t-\tau)) \rangle - \frac{1}{2}\langle \cos(2\omega(t-2\tau)) \rangle \right] \]

\[ r \begin{cases} 0 & \text{for } t = \tau \\ \frac{T_0}{8} & \text{for } t = 2\tau \\ \frac{T_0}{4} & \text{otherwise} \end{cases} \]

\[ \Rightarrow \text{echo depth} = T(2\tau) - \frac{T_0}{4} = \frac{T_0}{8} \]
$T(t)$ and $C_{TT}(t)$

It can be shown that:

$$\langle \cos(2\omega t) \rangle = \frac{\langle \delta T(t) \delta T(0) \rangle}{\langle \Delta T^2 \rangle} = C_{TT}(t), \quad \delta T(t) = T(t) - \langle T \rangle$$

\[ \downarrow \]

$$T(t) = \frac{T_0}{4} \left[ 1 + C_{TT}(\tau) - C_{TT}(t-\tau) - \frac{1}{2} C_{TT}(t) - \frac{1}{2} C_{TT}(|t-2\tau|) \right]$$

$$\approx \frac{T_0}{4} \left[ 1 - \frac{1}{2} C_{TT}(|t-2\tau|) \right] \quad \text{for} \ t > \tau$$
T-Quench Echo: Harmonic Approximation

\[ T(t) \approx \frac{T_0}{2} \left( 1 - C_{TT}(t - \tau) - \frac{1}{2} C_{TT}(|t - 2\tau|) \right) \]

\[ C_{TT}(t) = \exp \left( -\frac{t}{\tau_0} \right), \quad \tau_0 \approx 2.2 \text{ fs} \]
Dephasing Time of T-Quench Echoes

\[ \Delta T(\tau) = \Delta T(0) \exp\left[ -\frac{\tau}{\tau_{\text{dephase}}} \right] \]
Can we get temperature echo(es) by reassigning the same set of atomic velocities (corresponding to $T_0$!) at $t = 0$ and $t = \tau$?

$$\nu_i(0^+) = \nu_i(\tau^+) = u_i, \ i = 1, \ldots, 3N - 6$$

Answer: YES!

$$T(t) \approx T_0 \left[ 1 - \frac{1}{2} C_{\tau\tau}(|t - 3\tau/2|) \right]$$
Is it possible to produce temperature echo with a single velocity reassignment?

Reset all velocities at time $t$ to the values at a previous instant of time, i.e., $t = 0$. 

![Graph showing temperature vs. time with a sharp peak at 200 fs]