



# **Hands-on Workshop on Computational Biophysics**

by

**The Theoretical and Computational Biophysics Group  
(TCBG)**

and

**The National Center for Multiscale Modeling of  
Biological Systems (MMBioS)**



# ProDy

Protein Dynamics Analysis in Python



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Assist Prof, DCSB, Pitt

## Reference:

Bakan A, Meireles LM, Bahar I. (2011) ProDy: Protein dynamics inferred from theory and experiments *Bioinformatics* **27**:1575-7  
Bakan,A., Dutta,A., Whenzi, M., Liu,Y., Chennubhotla, C., Lezon,T.R., & Bahar, I. (2014) *Bioinformatics* in press.



# ProDy for exploring conformational space

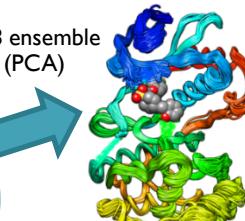
Protein Dynamics Analysis in Python

User inputs a protein sequence

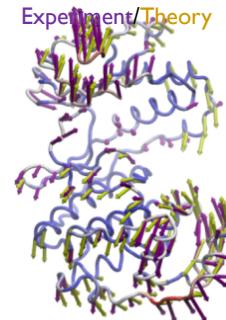
>IA9U:A|PDBID|CHAIN  
GSSHHHHHHSSGLVPRGSHMSQERP  
TFYRQELNKTIVEVPERYQNLSPVG  
SGAYGSVCAAFDTKTRGLRAVKKLS  
RPFQSIIHAKRTYRELRLKKHMKHEN  
VIGLLDVFT.....

ProDy identifies, retrieves, aligns, and analyzes (PCA) structures that match the input sequence

p38 ensemble (PCA)



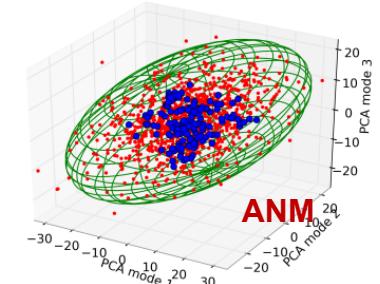
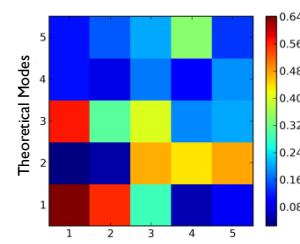
Experiment/Theory



p38 network model (ANM)

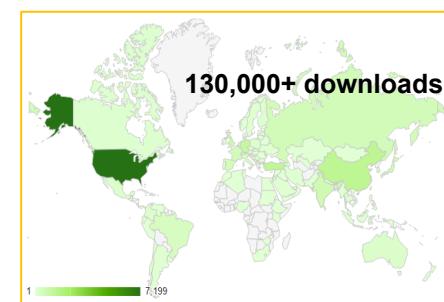
User can compare experimental and theoretical models

Overlap table

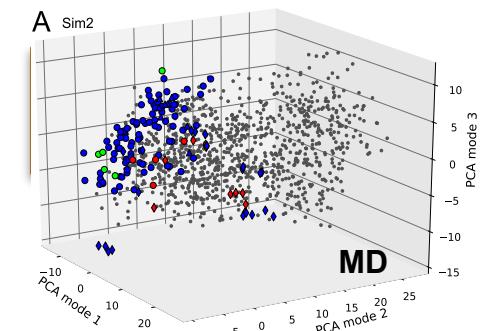


## Growth of Source Code and Usage

	Releases	Downloads	Visits <sup>2</sup>	Unique <sup>3</sup>
Nov '10 - Oct '11	19	8,530	8,678	2,946
Nov '11 - Oct '12	15	35,108	16,472	6,414
Nov '12 - Oct '13	8*	87,909	19,888	8,145
<b>Total</b>	<b>42</b>	<b>131,547</b>	<b>45,038</b>	<b>17,505</b>



Source <http://www.google.com/analytics/>

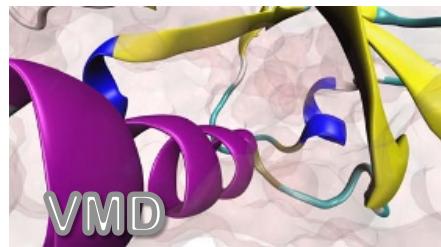
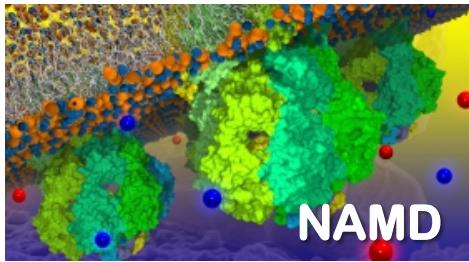


Bakan & Bahar, PSB 2011, 181-192

# Tutorials

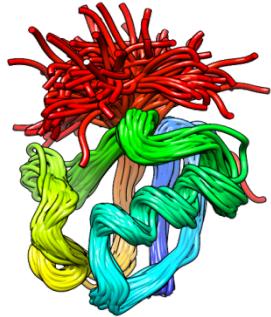
**Days 1-3**

<http://www.ks.uiuc.edu/Training/Tutorials/>

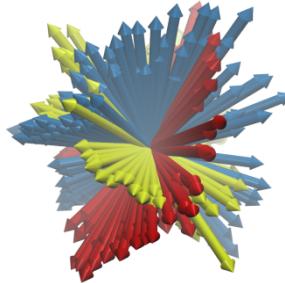


**Days 4-5**

<http://www.csb.pitt.edu/prody/#tutorials>



ProDy



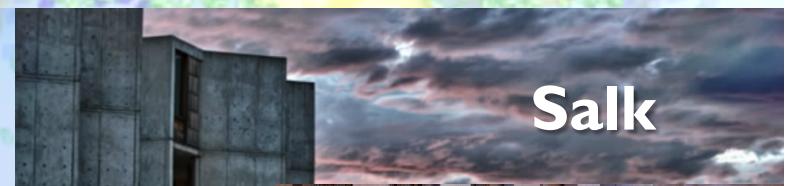
NMWiz

**Biomedical Technology Research Center (BTRC)**

**High Performance Computing  
for  
Multiscale Modeling of Biological Systems**

*Overarching biological theme:*

- Spatial organization
  - Temporal evolution
- of (neuro)signaling systems/events



Salk



PSC



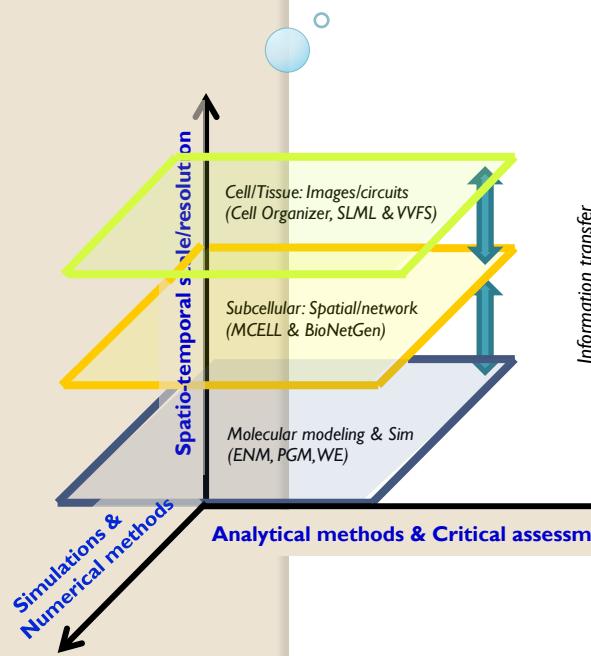
Acknowledgment: NIH - 5 P41 GM10371202



CMU

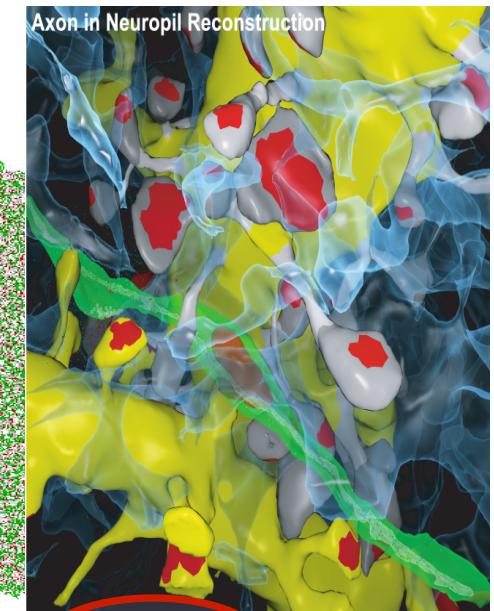
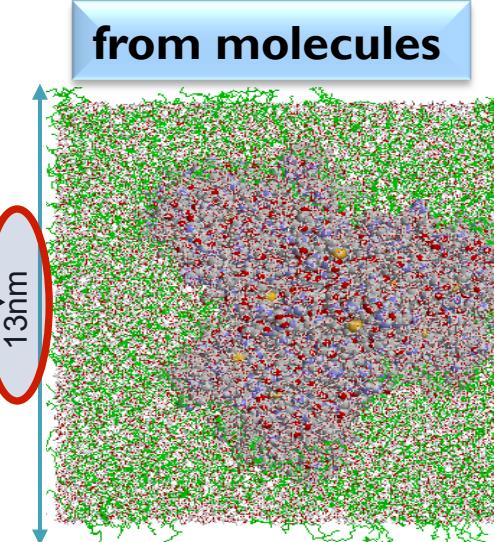
# GOAL: TO GENERATE DATA FOR MESOSCOPIC SCALE

**Developing integrated** methodology to enable information transfer across scales



**Microphysiological simulations**

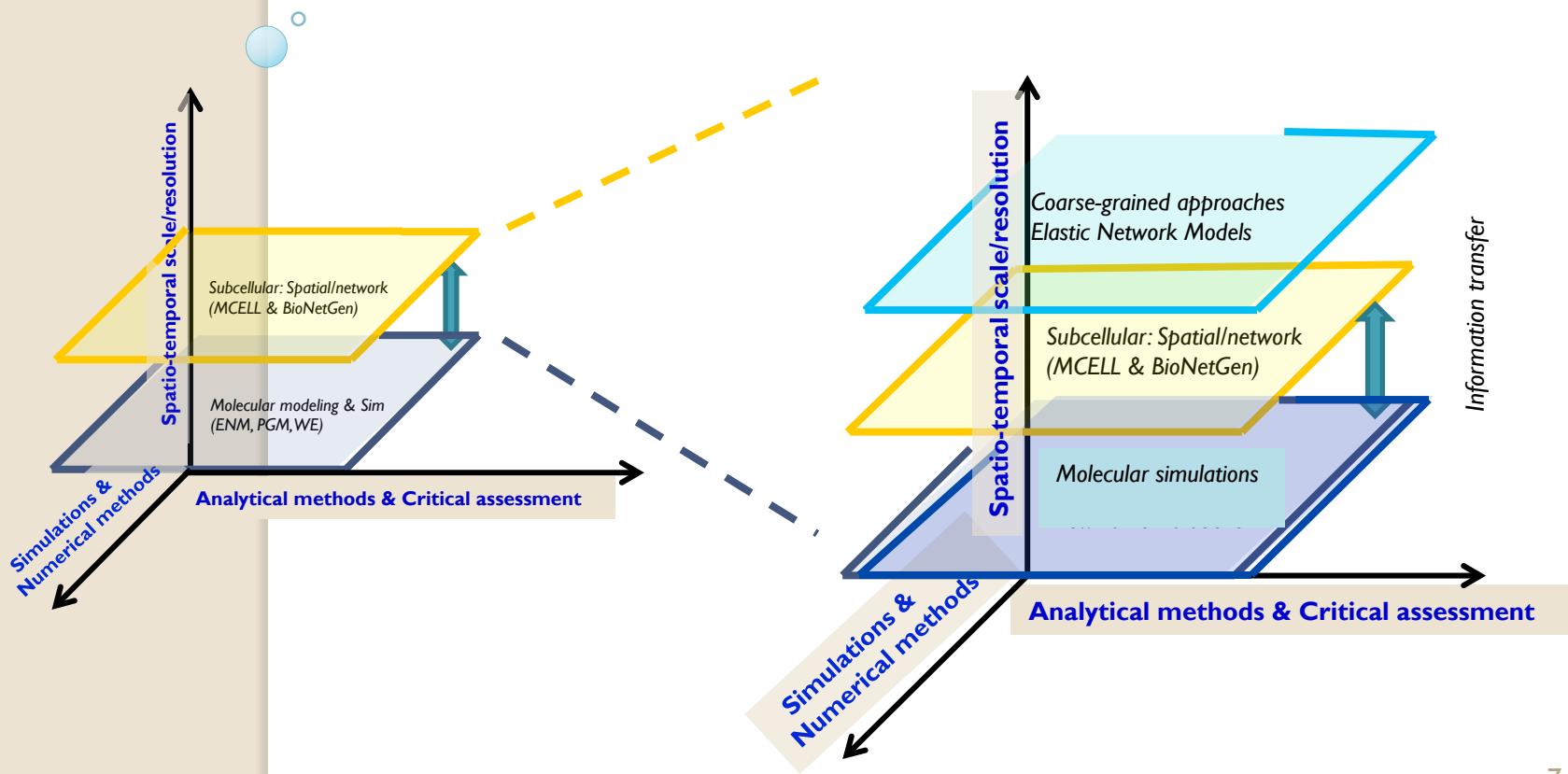
**to subcellular events**



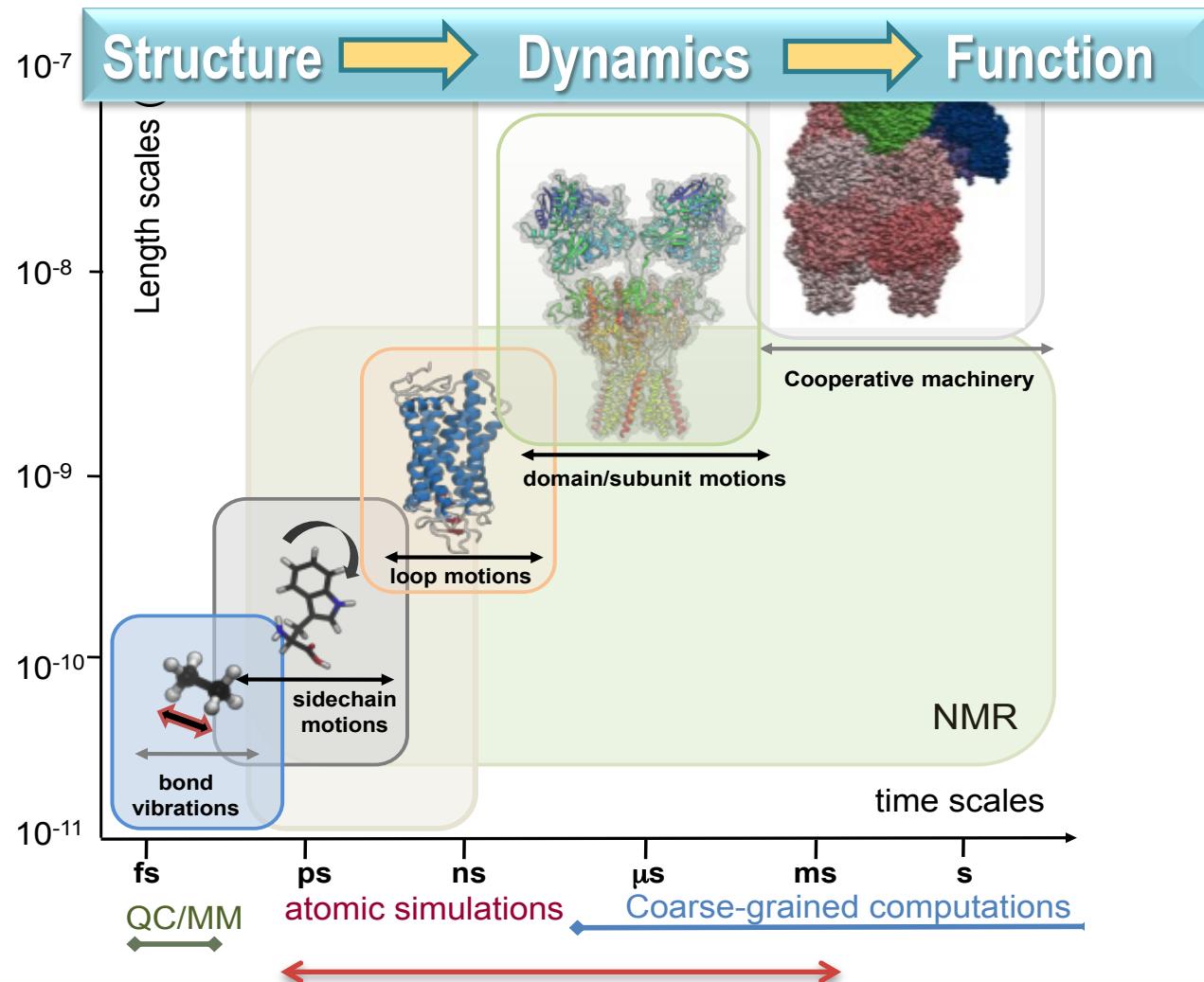
from  $6 \times 6 \times 5 \mu\text{m}^3$  sample of adult rat hippocampal stratum radiatum neuropil

# Goal: to generate data for mesoscopic scale

**Developing integrated** methodology for complex systems dynamics, to enable information transfer across scales



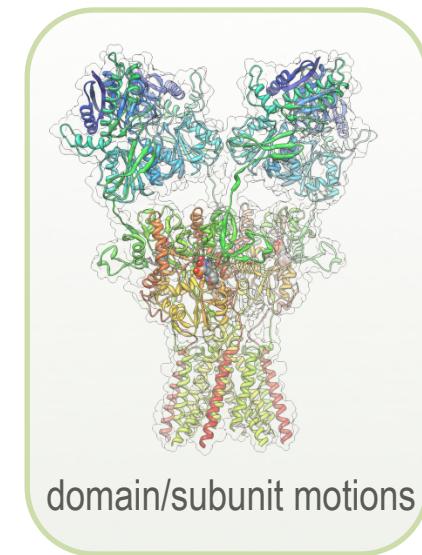
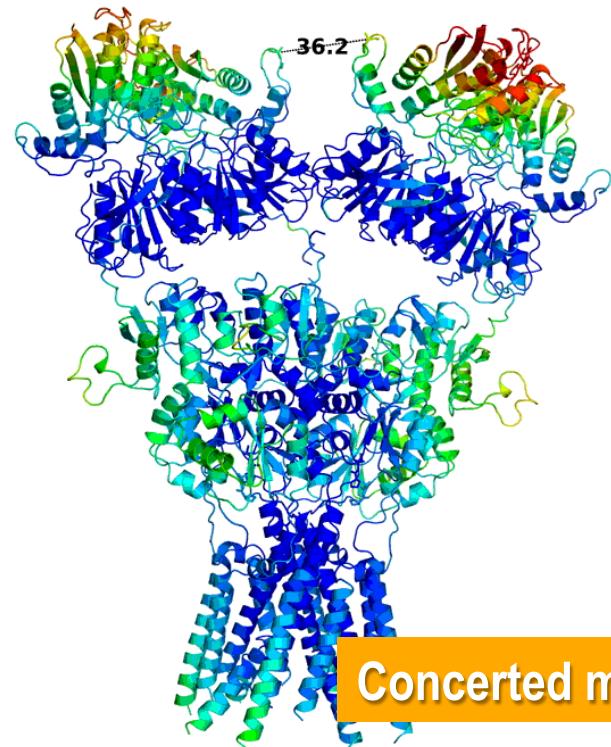
# Each structure encodes a unique dynamics



# Each structure encodes a **unique** dynamics

Structure → Dynamics → Function

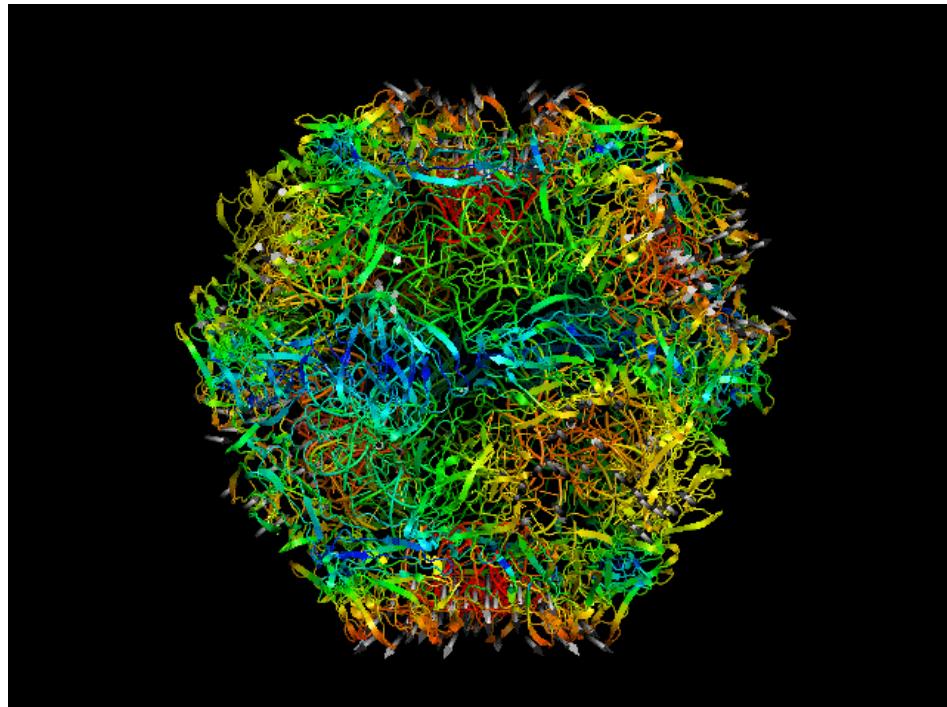
Signaling dynamics of AMPARs and NMDARs



Concerted movements of signaling molecules

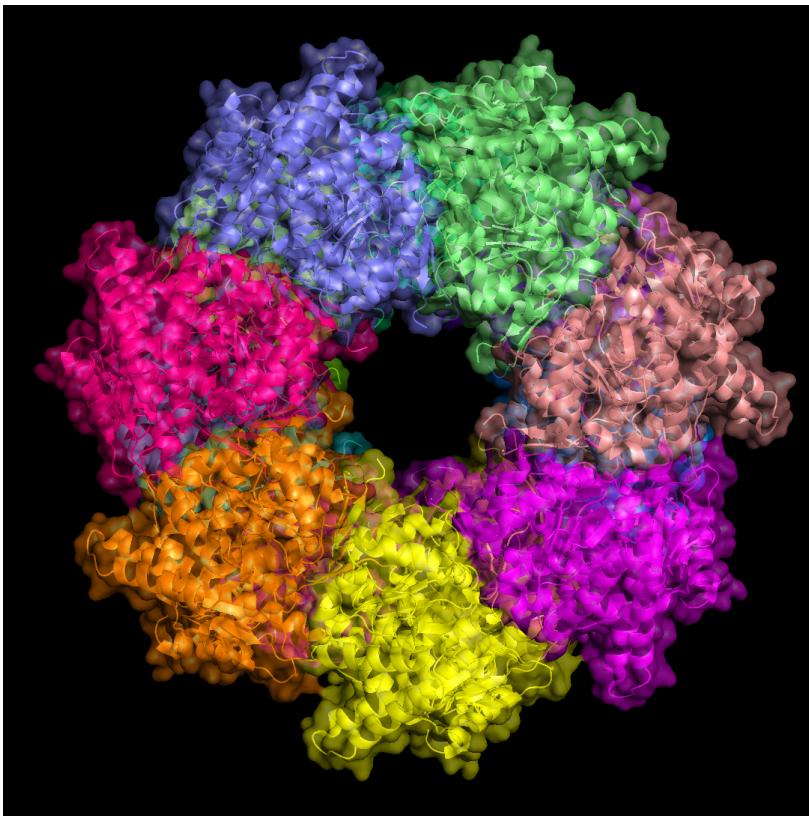
# Many proteins are molecular machines

And mechanical properties become more important in complexes/assemblies



STMV dynamics (Zheng Yang)

# Representation of structure as a network



## Why network models?

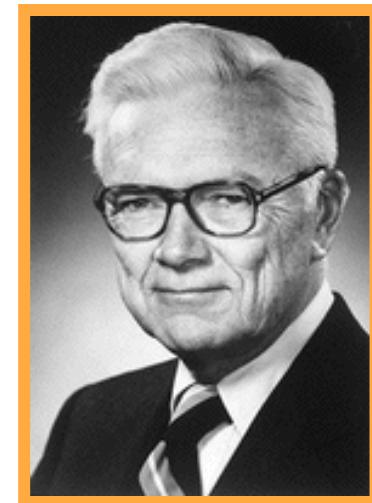
- for large systems' collective motions & long time processes beyond the capability of full atomic simulations
- to incorporate structural data in the models – at multiple levels of resolution
- to take advantage of theories of polymer physics, spectral graph methods, etc.

# Physics-based approach

- Statistical Mechanics of Polymers
- Theory of Rubber Elasticity



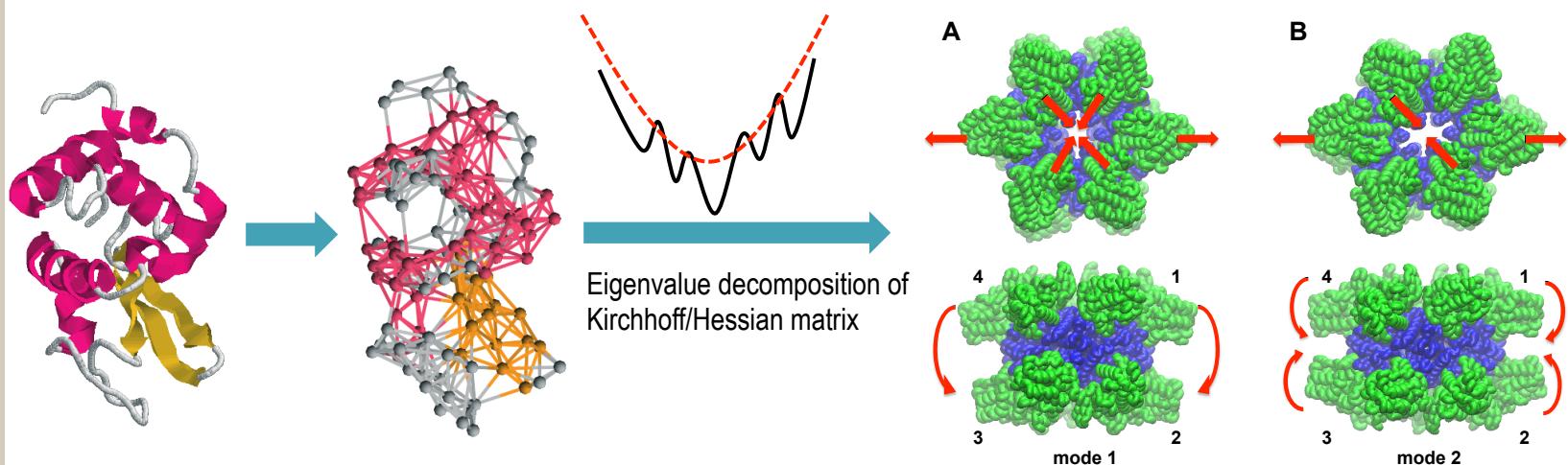
**Elastic Network Model for Proteins**



Paul J. Flory (1910-1985)  
Nobel Prize in Chemistry 1974

And Pearson (1976), Eichinger (1980), Klockzkowski, Erman & Mark (1989)...

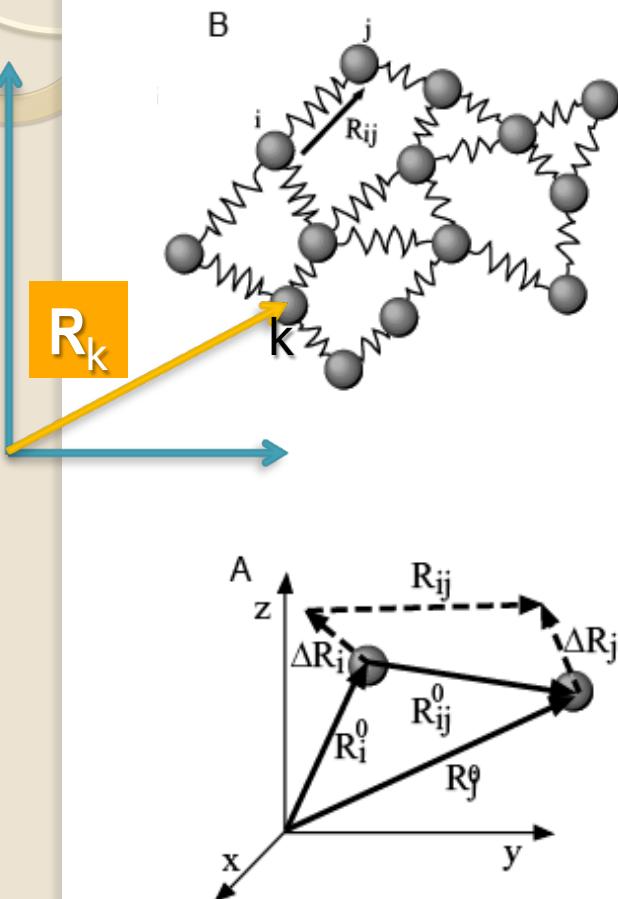
# Collective motions using elastic network models (ENM)



**GNM:** Bahar et al *Fold & Des* 1996; Haliloglu et al. *Phys Rev Lett* 1997  
**ANM:** Doruker et al. *Proteins* 2000; Atilgan et al, *Biophys J* 2001

Based on theory of elasticity for  
polymer networks by **Flory, 1976**

# Gaussian network model (GNM)



- Each node represents a residue
- Residue positions,  $\mathbf{R}_i$ , identified by their  $\alpha$ -carbons' coordinates
- Springs connect residues located within a cutoff distance (e.g., 10 Å)

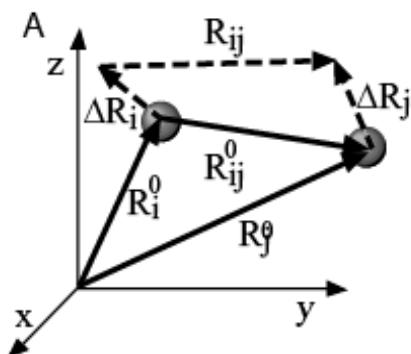
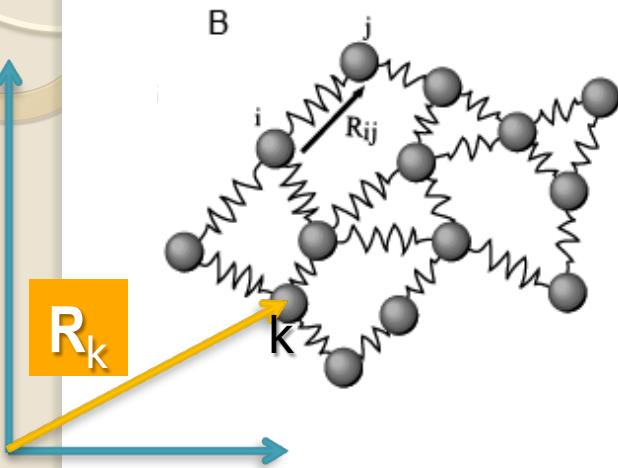
→ Nodes are subject to **Gaussian fluctuations**  $\Delta\mathbf{R}_i$

→ Inter-residue distances  $R_{ij}$  also undergo Gaussian fluctuations

$$\rightarrow \Delta\mathbf{R}_{ij} = \Delta\mathbf{R}_j - \Delta\mathbf{R}_i$$

**Fluctuations in residue positions**

# Gaussian network model (GNM)



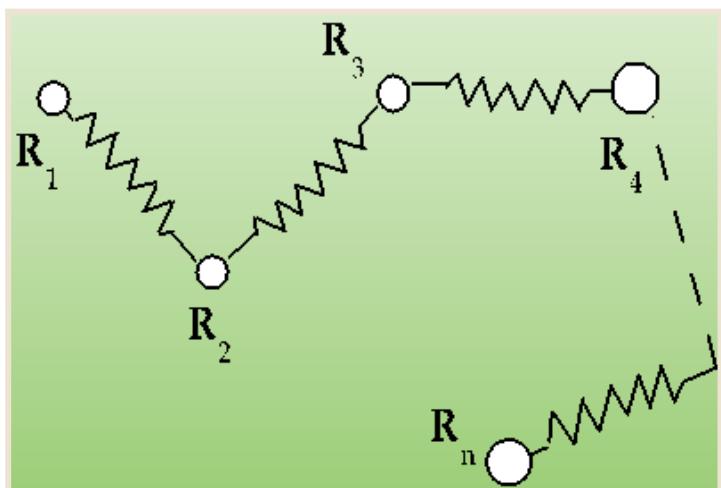
Fluctuation vector:

$$\rightarrow \Delta \mathbf{R} = \begin{bmatrix} \Delta \mathbf{R}_1 \\ \Delta \mathbf{R}_2 \\ \Delta \mathbf{R}_3 \\ \Delta \mathbf{R}_4 \\ \vdots \\ \vdots \\ \vdots \\ \Delta \mathbf{R}_N \end{bmatrix}$$

Fluctuations in residue positions

# Rouse model for polymers

Classical bead-and-spring model



Kirchhoff matrix

$$\Gamma = \begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & .. & .. & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix}$$

Force constant  $\Delta R_{12} = R_{12} - R_{12}^0$

$$V_{\text{tot}} = (\gamma/2) [ (\Delta R_{12})^2 + (\Delta R_{23})^2 + \dots + (\Delta R_{N-1,N})^2 ]$$
$$= (\gamma/2) [ (\Delta R_2 - \Delta R_1)^2 + (\Delta R_3 - \Delta R_2)^2 + \dots ]$$

# Rouse model for polymers

Kirchhoff matrix

$$\Gamma = \begin{bmatrix} 1 & -1 & & \\ -1 & 2 & -1 & \\ & -1 & 2 & -1 \\ & & \ddots & \ddots \\ & & & -1 & 2 & -1 \\ & & & & -1 & 1 \end{bmatrix}$$

Force constant

$$\begin{aligned} V_{\text{tot}} &= (\gamma/2) [ (\Delta R_{12})^2 + (\Delta R_{23})^2 + \dots + (\Delta R_{N-1,N})^2 ] \\ &= (\gamma/2) [ (\Delta R_2 - \Delta R_1)^2 + (\Delta R_3 - \Delta R_2)^2 + \dots ] \end{aligned}$$

# Rouse model for polymers

Fluctuation vector      Kirchhoff matrix

$$(\gamma/2) [\Delta\mathbf{R}_1 \ \Delta\mathbf{R}_2 \ \Delta\mathbf{R}_3 \ \dots \ \Delta\mathbf{R}_N] \begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & \ddots & \ddots & \\ & & & -1 & 2 & -1 \\ & & & & 1 & 1 \end{bmatrix} = [\Delta\mathbf{R}_1 \ \Delta\mathbf{R}_2 \ \Delta\mathbf{R}_3]$$

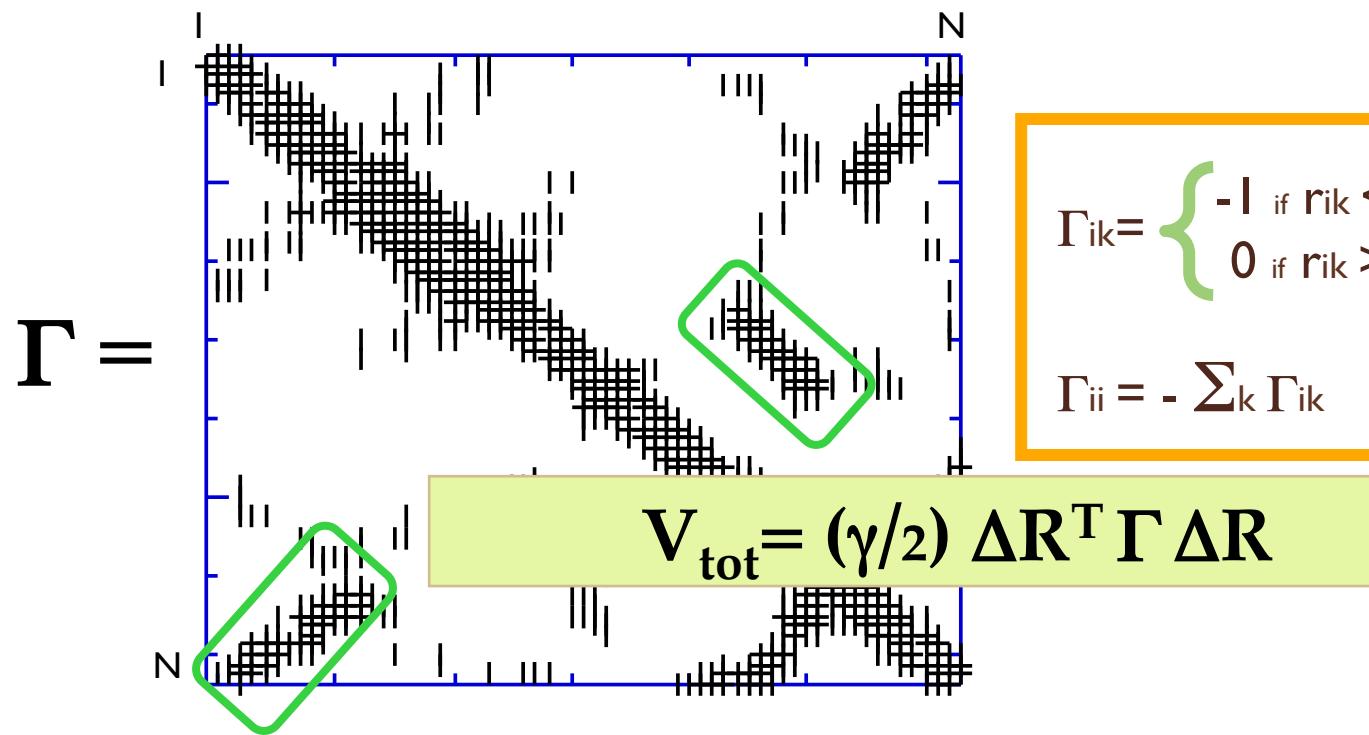
Force constant

$$\boxed{\mathbf{V}_{\text{tot}} = (\gamma/2) \Delta\mathbf{R}^T \Gamma \Delta\mathbf{R}}$$

$$\begin{aligned} \mathbf{V}_{\text{tot}} &= (\gamma/2) [ (\Delta\mathbf{R}_{12})^2 + (\Delta\mathbf{R}_{23})^2 + \dots + (\Delta\mathbf{R}_{N-1,N})^2 ] \\ &= (\gamma/2) [ (\Delta\mathbf{R}_2 - \Delta\mathbf{R}_1)^2 + (\Delta\mathbf{R}_3 - \Delta\mathbf{R}_2)^2 + \dots ] \end{aligned}$$

# Kirchhoff matrix for inter-residue contacts

For a protein of  $N$  residues



$\Gamma$  provides a complete description of contact topology!

# Statistical mechanical averages

For a protein of N residues

$$\langle \Delta \mathbf{R}_i \cdot \Delta \mathbf{R}_j \rangle = (1/Z_N) \int (\Delta \mathbf{R}_i \cdot \Delta \mathbf{R}_j) e^{-V/k_B T} d\{\Delta \mathbf{R}\}$$

$$= (3 k_B T / \gamma) [\Gamma^{-1}]_{ij}$$

$\Gamma$  provides a complete description of contact topology!



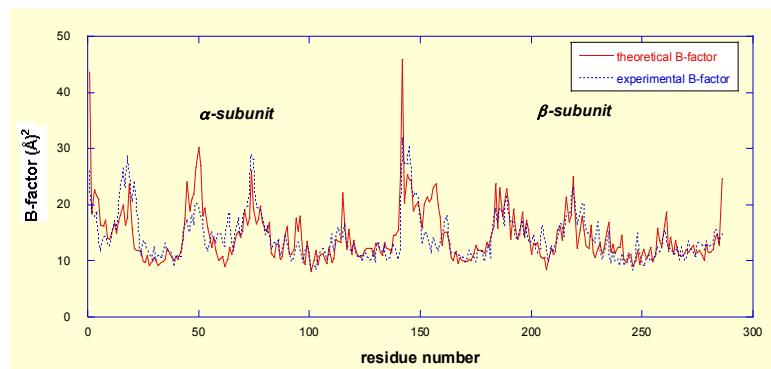
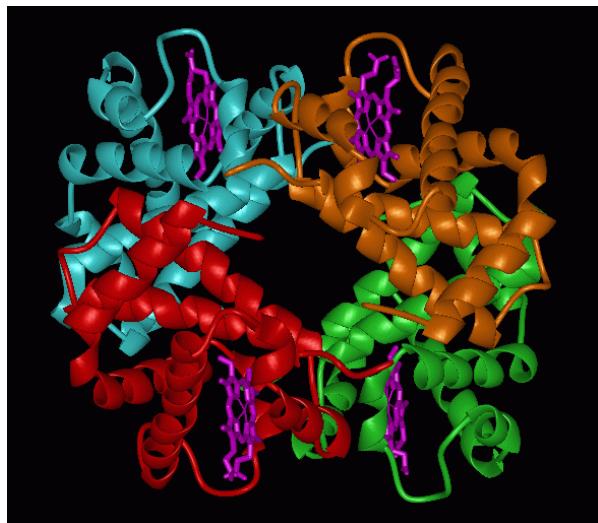
Kirchhoff matrix determines the mean-square fluctuations

$$[\Gamma^{-1}]_{ii} \sim \langle (\Delta\mathbf{R}_i)^2 \rangle$$

And cross-correlations between residue motions

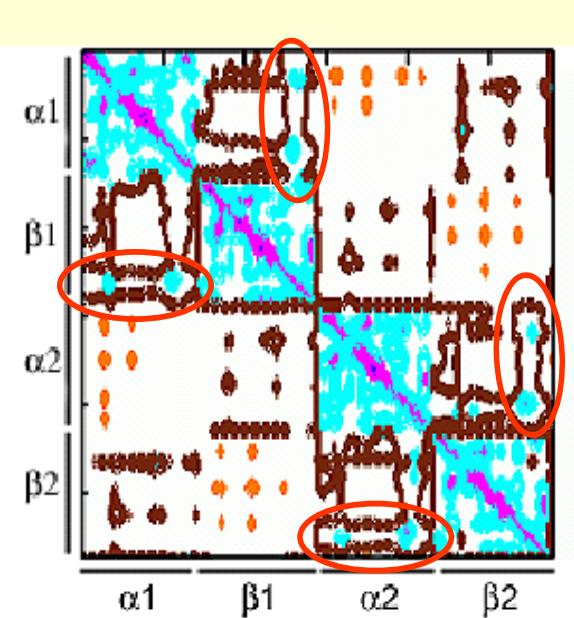
$$[\Gamma^{-1}]_{ij} \sim \langle (\Delta\mathbf{R}_i \cdot \Delta\mathbf{R}_j) \rangle$$

# 1. Application to hemoglobin



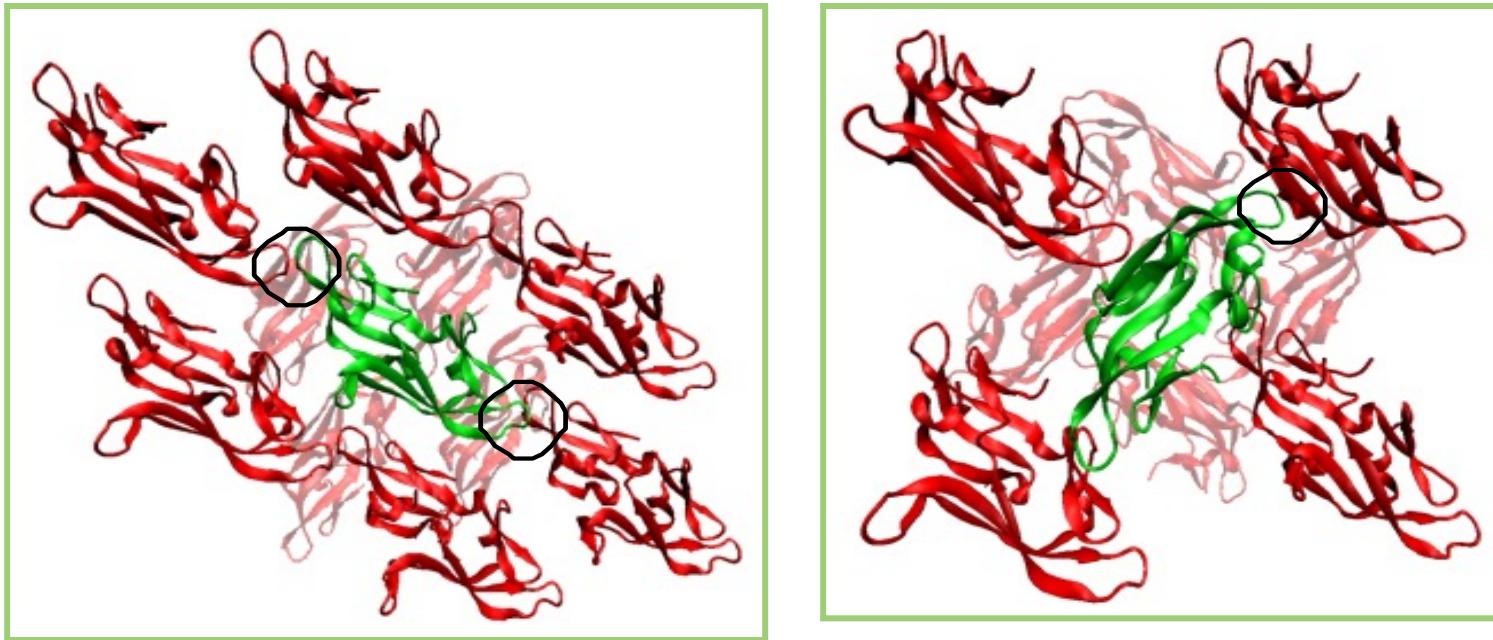
B-factors – Comparison with experiments

$$B_i = 8\pi^2/3 <(\Delta R_i)^2>$$



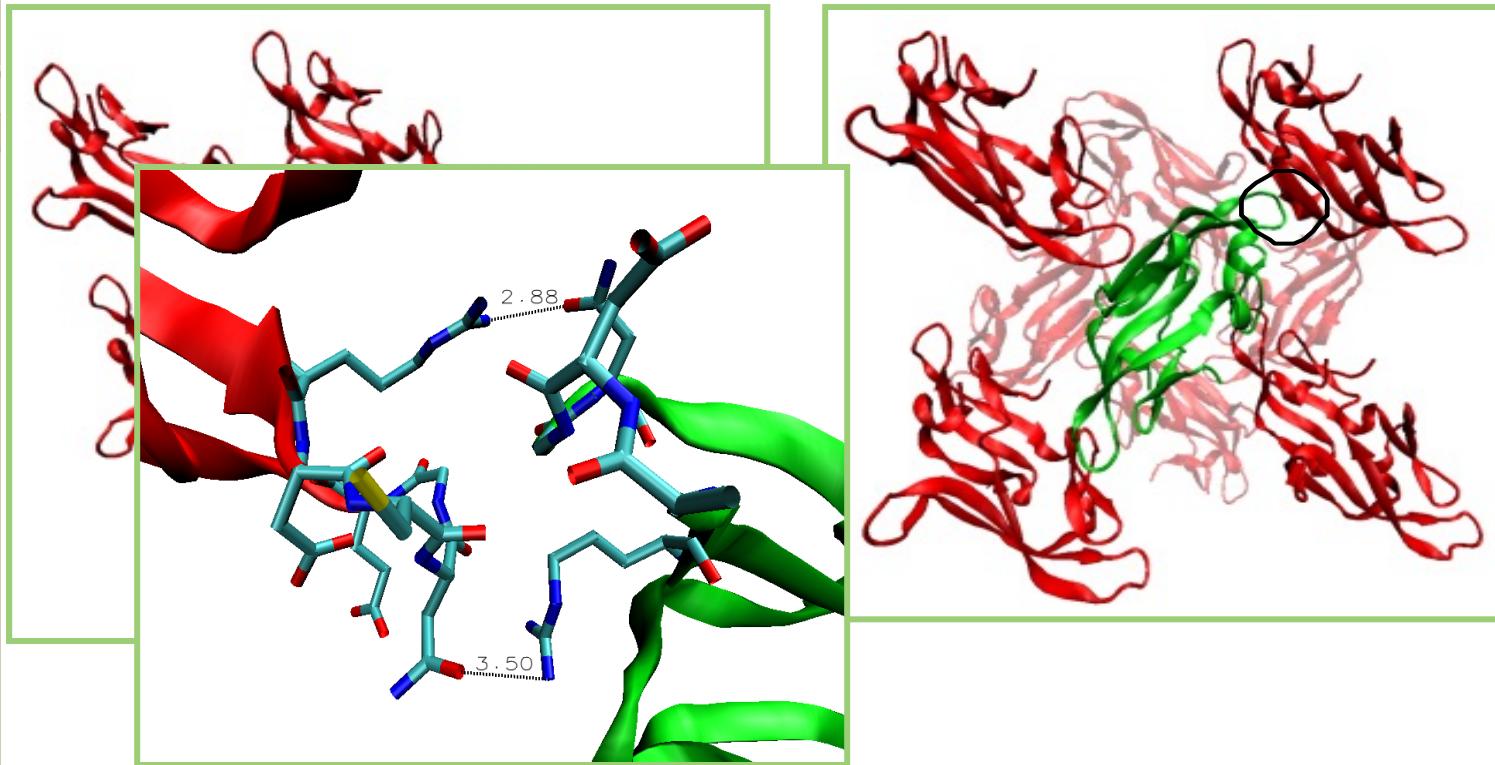
Intradimer cooperativity – Symmetry rule  
(Yuan et al. JMB 2002; Ackers et al. PNAS 2002.)

# B-factors are affected by crystal contacts



Two X-ray structures for a designed sugar-binding protein LKAMG

# B-factors are affected by crystal contacts

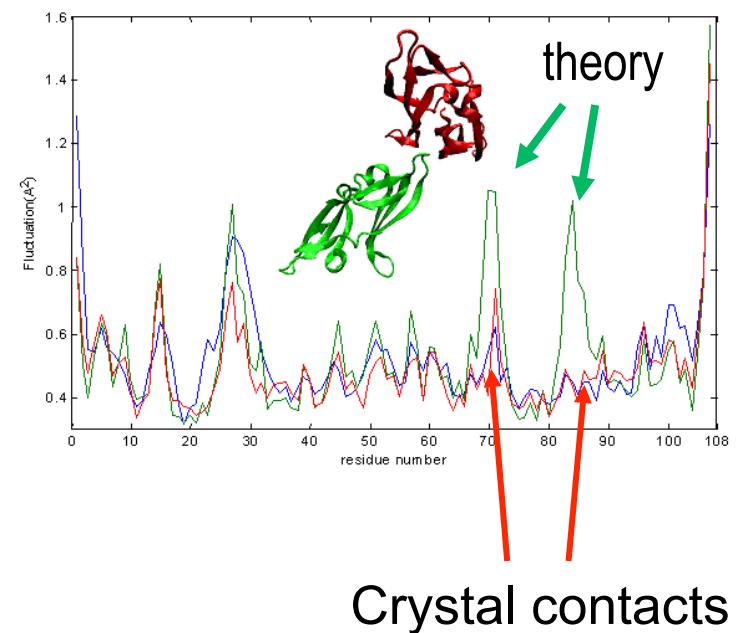
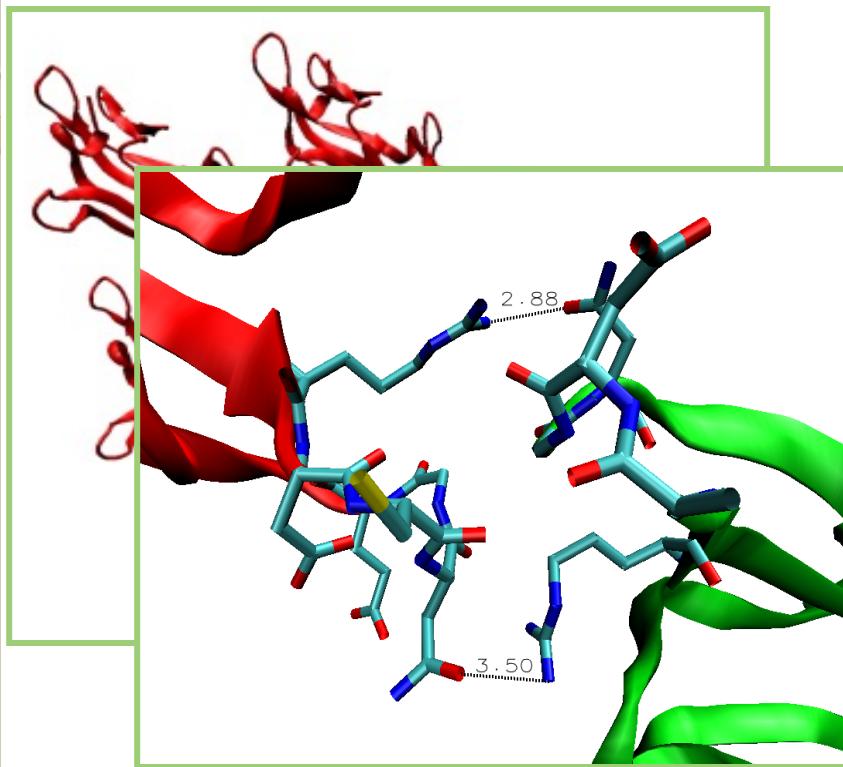


Particular loop motions are curtailed by intermolecular contacts in the crystal environment causing a discrepancy between theory and experiments

FOR MORE INFO...

Liu, Koharudin, Gronenborn & Bahar (2009) *Proteins* 77, 927-939.

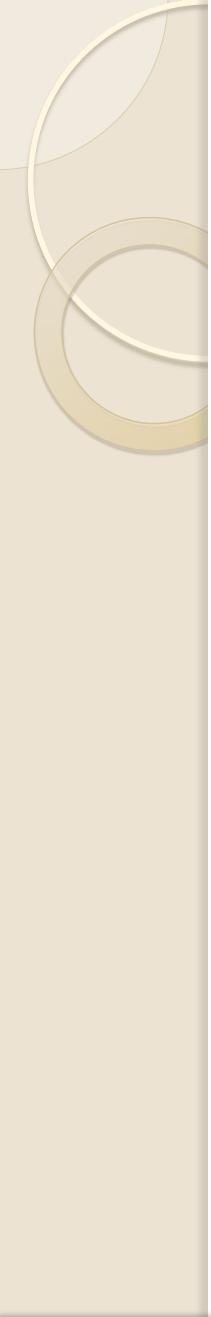
# Agreement between theory and experiments upon inclusion of crystal lattice effects into the GNM



Particular loop motions are curtailed by intermolecular contacts in the crystal environment causing a discrepancy between theory and experiments

FOR MORE INFO...

Liu, Koharudin, Gronenborn & Bahar (2009) *Proteins* 77, 927-939.



# Collective Motions Encoded by the Structure: **Normal Modes**

# Several modes contribute to dynamics

$$\langle \Delta\mathbf{R}_i \cdot \Delta\mathbf{R}_j \rangle = \sum_k [\Delta\mathbf{R}_i \cdot \Delta\mathbf{R}_j]_k$$

Contribution of mode k

$$\langle \Delta\mathbf{R}_i \cdot \Delta\mathbf{R}_j \rangle = (3k_B T / \gamma) [\Gamma^{-1}]_{ij}$$

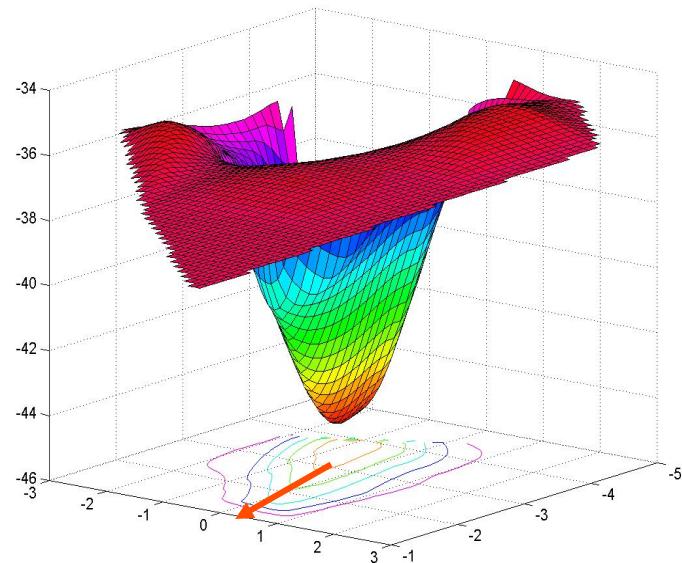
Contribution of mode k

$$[\Delta\mathbf{R}_i \cdot \Delta\mathbf{R}_j]_k = (3k_B T / \gamma) [\lambda_k^{-1} \mathbf{u}_k \mathbf{u}_k^T]_{ij}$$

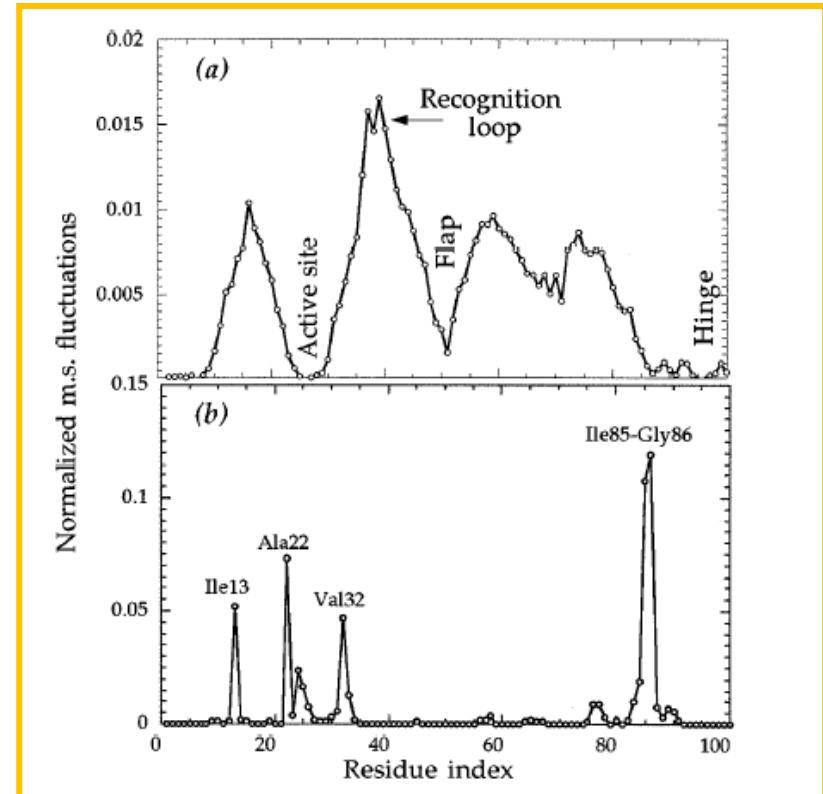
expressed in terms of kth eigenvalue  $\lambda_k$  and kth eigenvector  $\mathbf{u}_k$  of  $\Gamma$

FOR MORE INFO...

# Several modes contribute to dynamics



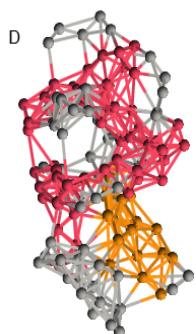
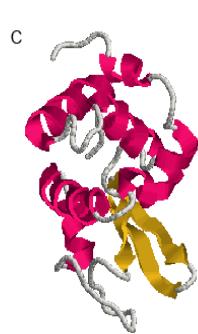
The first mode selects  
the 'easiest' collective motion



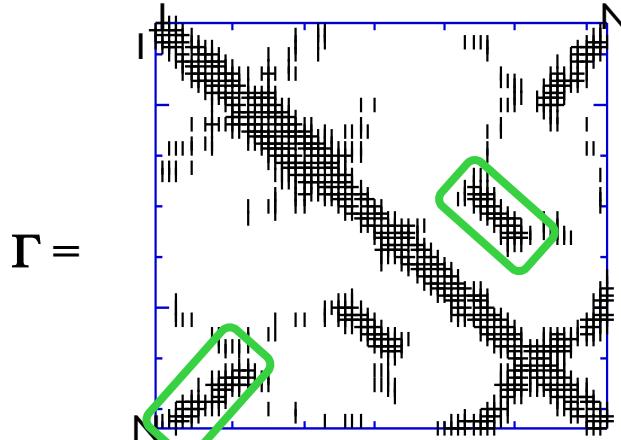
FOR MORE INFO...

Bahar et al. (1998) Phys Rev Lett. 80, 2733

# Gaussian network model (GNM)

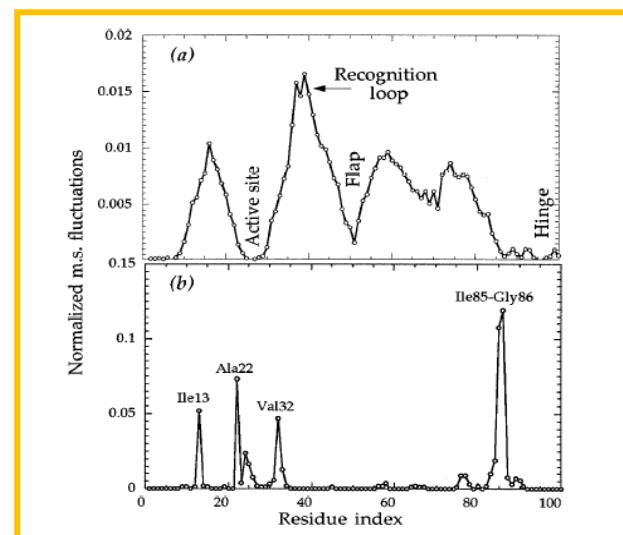


Kirchhoff matrix for inter-residue contacts



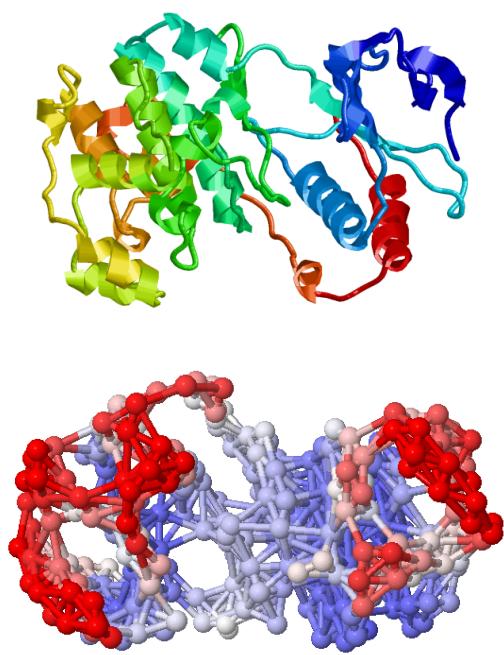
$$\langle \Delta\mathbf{R}_i \cdot \Delta\mathbf{R}_j \rangle = (1/Z_N) \int (\Delta\mathbf{R}_i \cdot \Delta\mathbf{R}_j) e^{-V/k_B T} d\{\Delta\mathbf{R}\} = (3k_B T / \gamma) [\Gamma^{-1}]_{ij}$$

$$[\Delta\mathbf{R}_i \cdot \Delta\mathbf{R}_i]_k = (3k_B T / \gamma) [\lambda_k^{-1} \mathbf{u}_k \mathbf{u}_k^T]_i$$



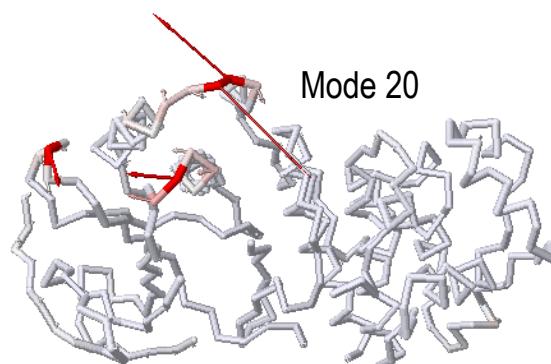
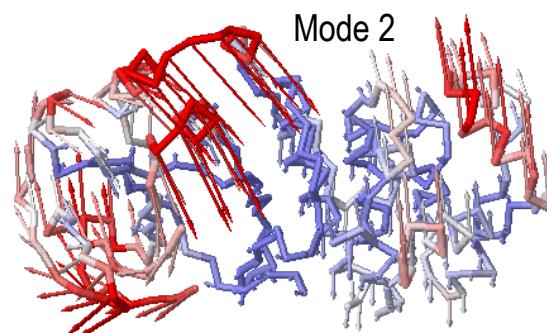
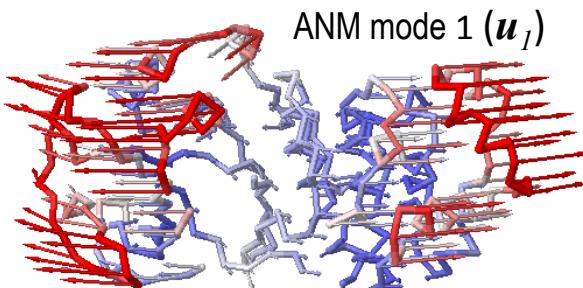
**Several modes of motion contribute to dynamics**

# Anisotropic Network Model (ANM)

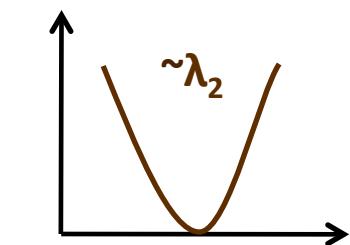
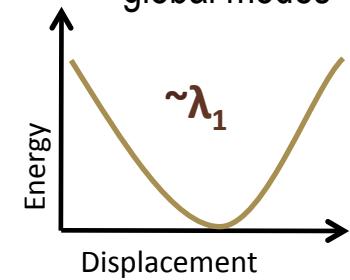


$$\mathbf{H} = \sum_{i=1}^{3N-6} \lambda_i \ \mathbf{u}_i \mathbf{u}_i^T$$

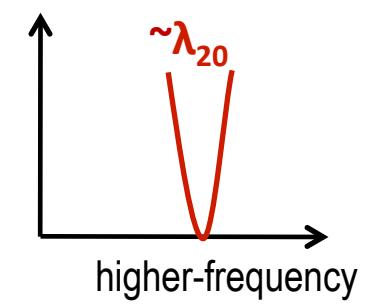
$$\mathbf{H}^{(ij)} = \frac{\gamma}{(R_{ij}^0)^2} \begin{bmatrix} X_{ij}X_{ij} & X_{ij}Y_{ij} & X_{ij}Z_{ij} \\ Y_{ij}X_{ij} & Y_{ij}Y_{ij} & Y_{ij}Z_{ij} \\ Z_{ij}X_{ij} & Z_{ij}Y_{ij} & Z_{ij}Z_{ij} \end{bmatrix}$$



low-frequency  
~ global modes



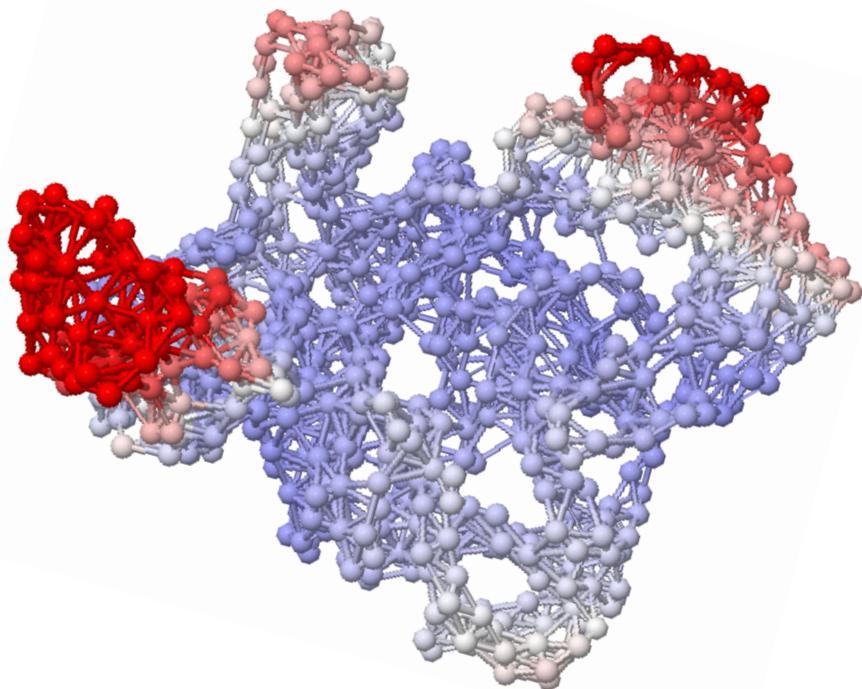
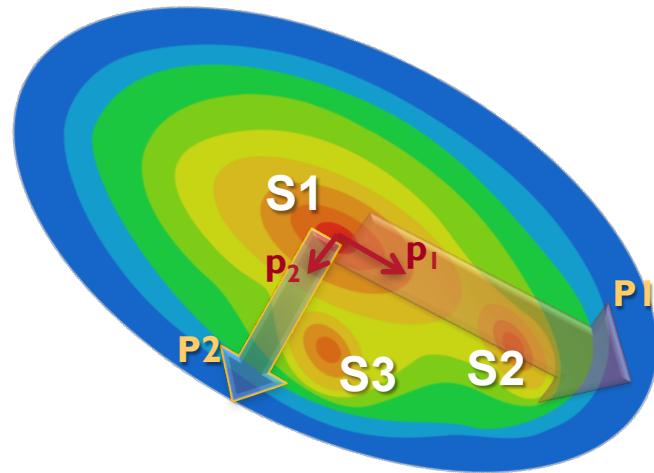
$\lambda_1 < \lambda_2 < \lambda_3 < \dots$



# Allosteric changes in conformation

## ANM (anisotropic network model)

Elastic Network Models are particularly useful for exploring the allosteric dynamics of large multimeric structures



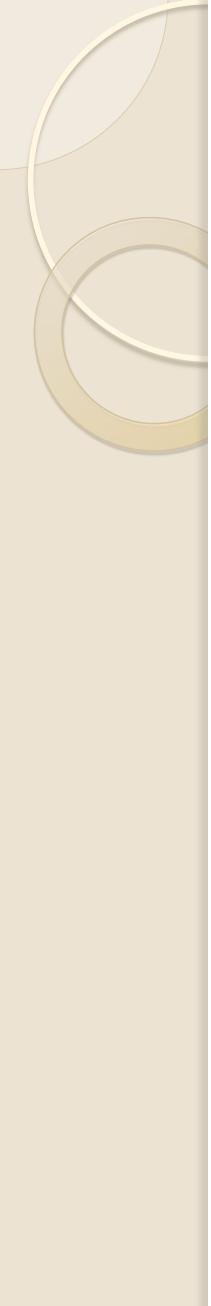
Comparison with experimental data shows that the functional movements are those predicted by the ENM to be intrinsically encoded by the structure

# Session I: Plotting $\langle(\Delta R_i)^2\rangle$ and contributions of selected modes

- from prody import \*
- from pylab import \*
- anm = calcANM('1cot', selstr='calpha')
- anm, cot = calcANM('1cot', selstr='calpha')
- anm
- cot
- figure()
- showProtein(cot)
  
- figure()
- showSqFlucts(anm)
  
- figure()
- showSqFlucts(anm[:10])
- 
- figure()
- showSqFlucts(anm[:10], label='10 modes')

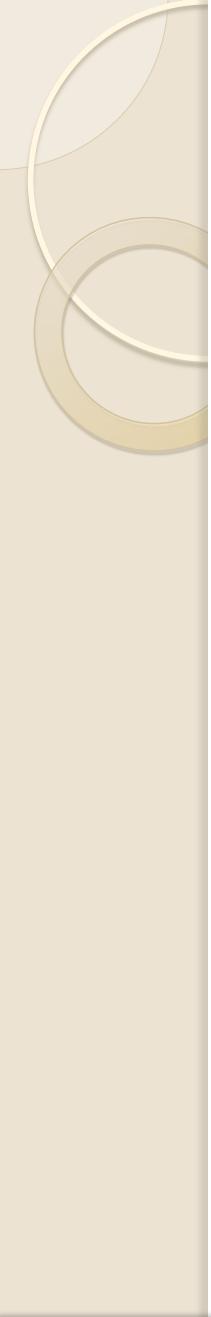
*Application to cytochrome c  
PDB: 1cot  
A protein of 121 residues*

cmd  
ipython



## Session 2: Viewing color-coded animations of individual modes

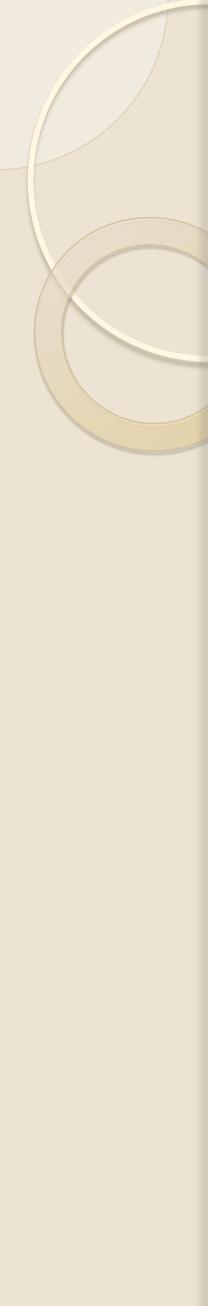
- `writeNMD('cot_anm.nmd', anm, cot)`
- *Start VMD*
- *select Extensions → Analysis → Normal Mode Wizard*
- *Select ‘Load NMD File’*



# Session 3: Cross-correlations

## $\langle(\Delta R_i \cdot \Delta R_j)\rangle$ between fluctuations

- cross\_corr = calcCrossCorr?
- cross\_corr = calcCrossCorr(anm[0])
- figure()
- showCrossCorr(anm[0])
- writeHeatmap('anm\_cross1.hm', cross\_corr)



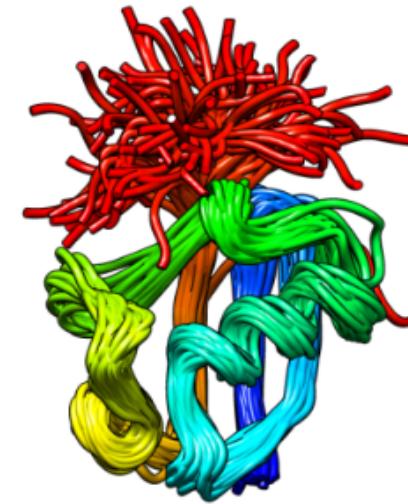
# Session 4:

## Viewing cross-correlations using VMD

- VMD – *Load file*
- Select *cot\_anm.nmd* (*from your local folder*)
- *Load HeatMap*
- *open anm\_cross1.hm* (*from your local folder*)

# Ensembles of structures

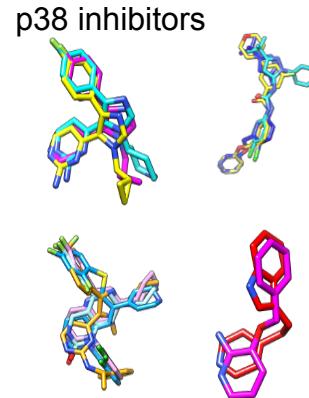
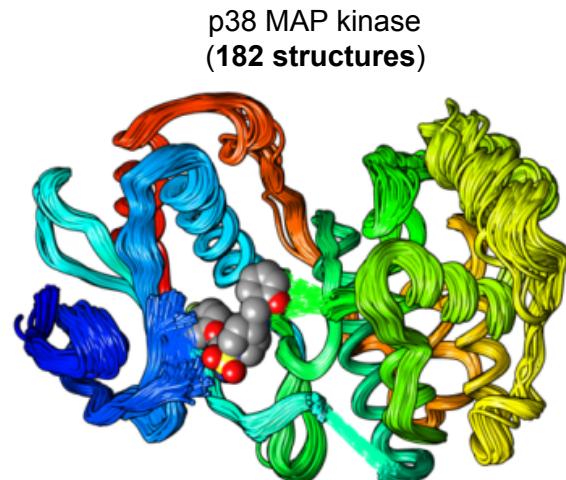
- Structural changes accompanying substrate (protein) binding
- Structural changes induced by, or stabilized upon, ligand binding



Ubiquitin  
140 structures  
1732 models

# Ensembles of structures

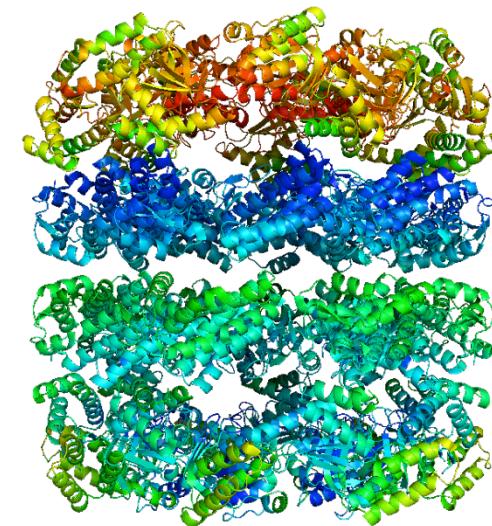
- Structural changes accompanying substrate (protein) binding
- Structural changes induced by, or stabilized upon, ligand binding



Ubiquitin  
**140 structures**  
**1732 models**

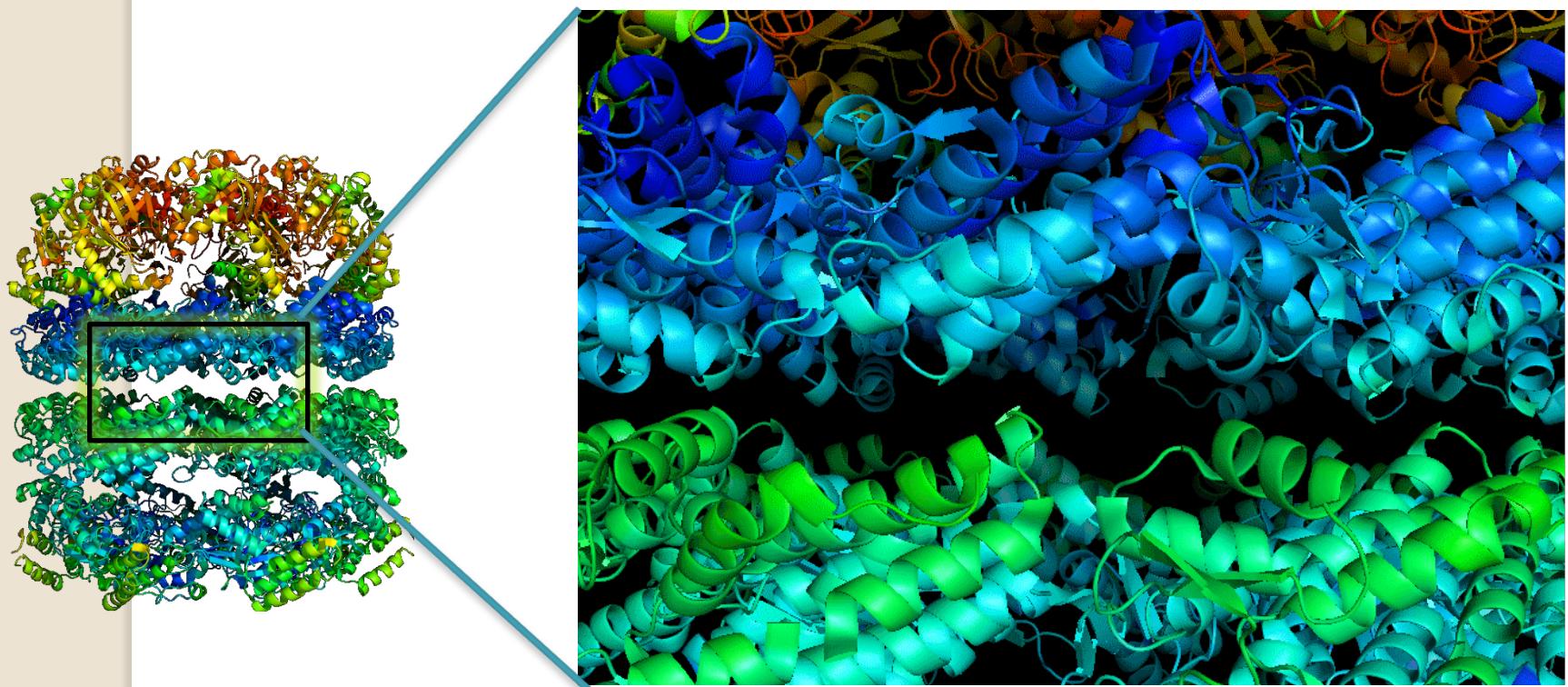
# Ensembles of structures

- Structural changes accompanying substrate (protein) binding
- Structural changes induced by, or stabilized upon, ligand binding
- Alternative conformations sampled during allosteric cycles

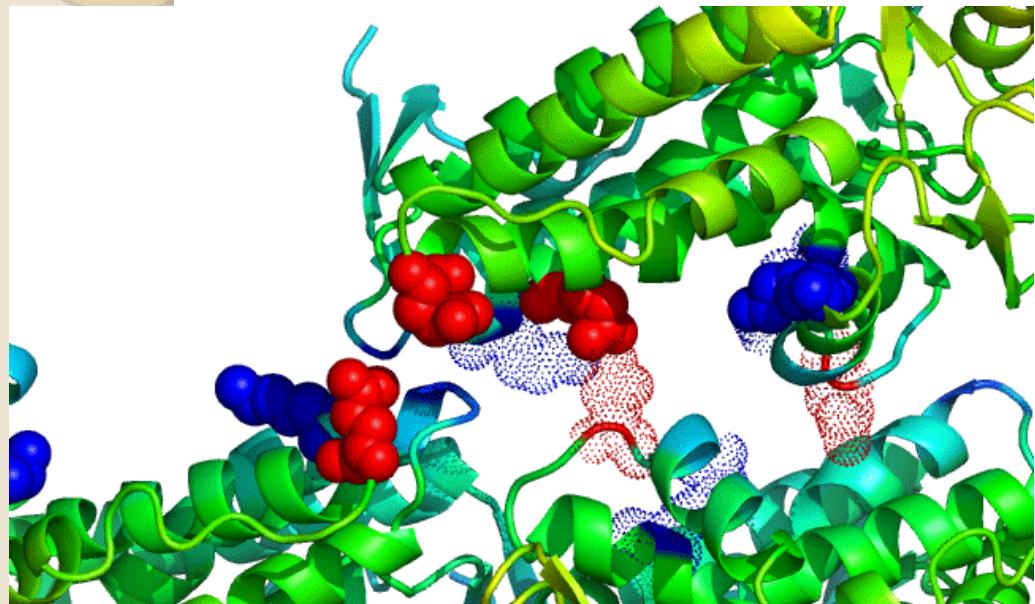


Yang et al. PLoS Comp Biol 2009

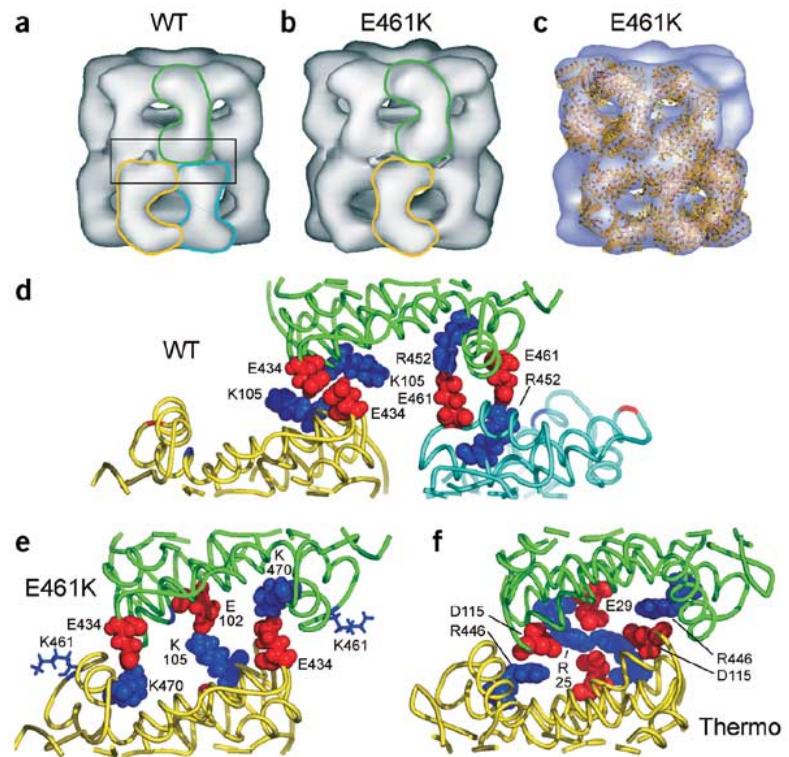
# Redistribution of interactions at interfaces



# Mutations may stabilize conformers along soft modes – which may be dysfunctional

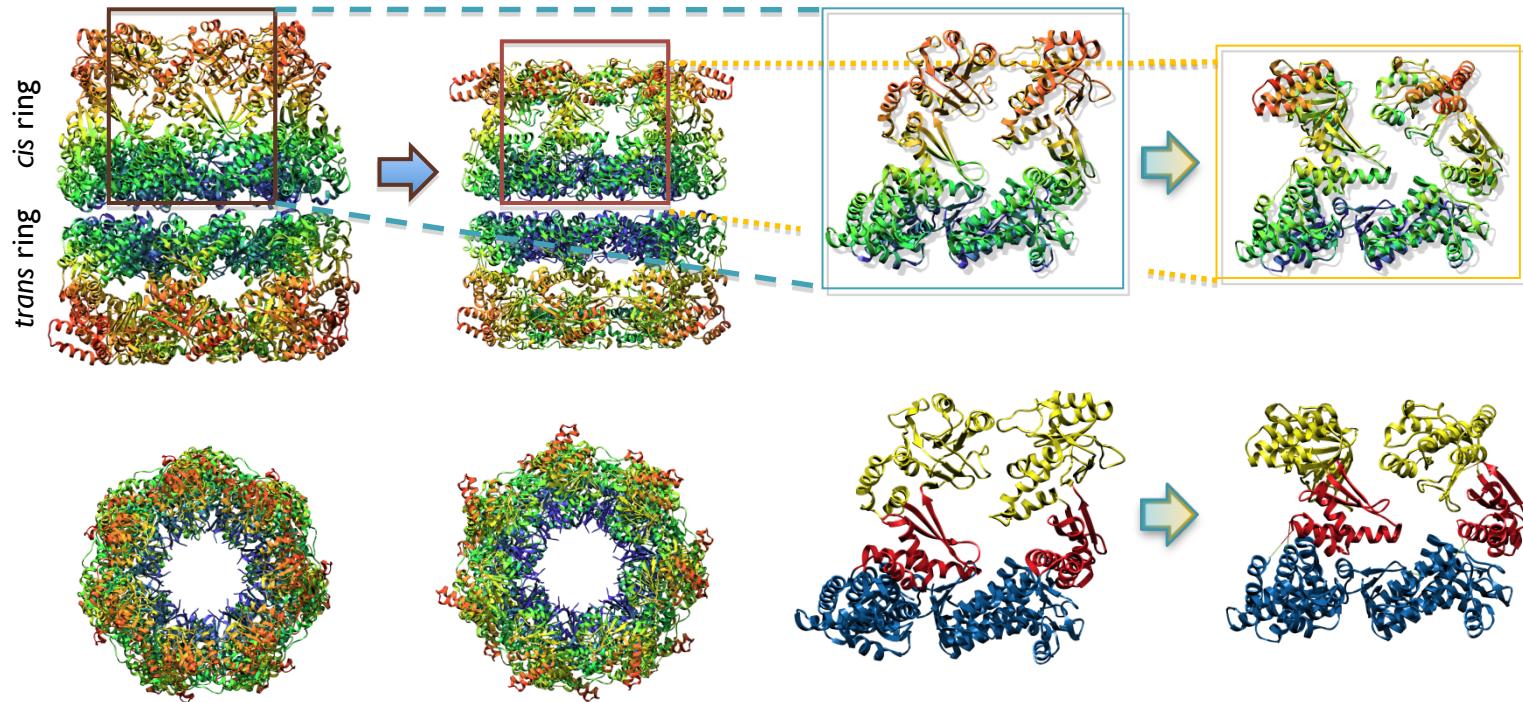


E461 mutant is a deformed structure along mode 1



E461K mutation causes disruption of inter-ring transfer of ATP-induced signal (Sewell et al NSB 2004)

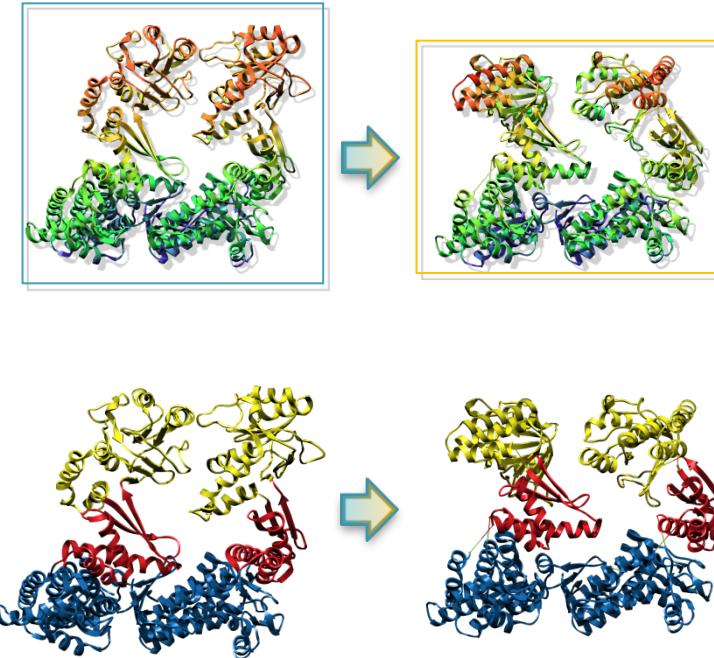
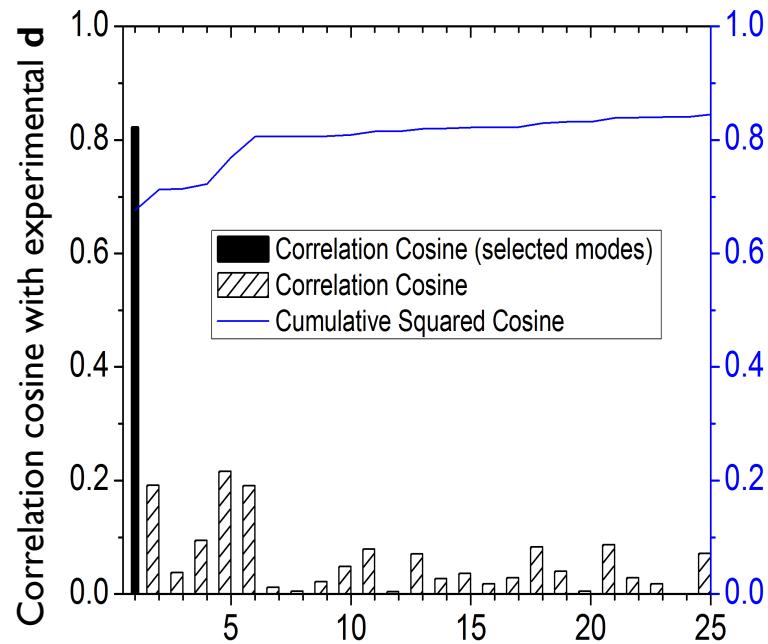
# Passage between the R and T state of GroEL



See...

Z Yang, P Marek and I Bahar, *PLoS Comp Biology* 2009

# The softest mode enables the passage R → T (with a correlation of 0.81)



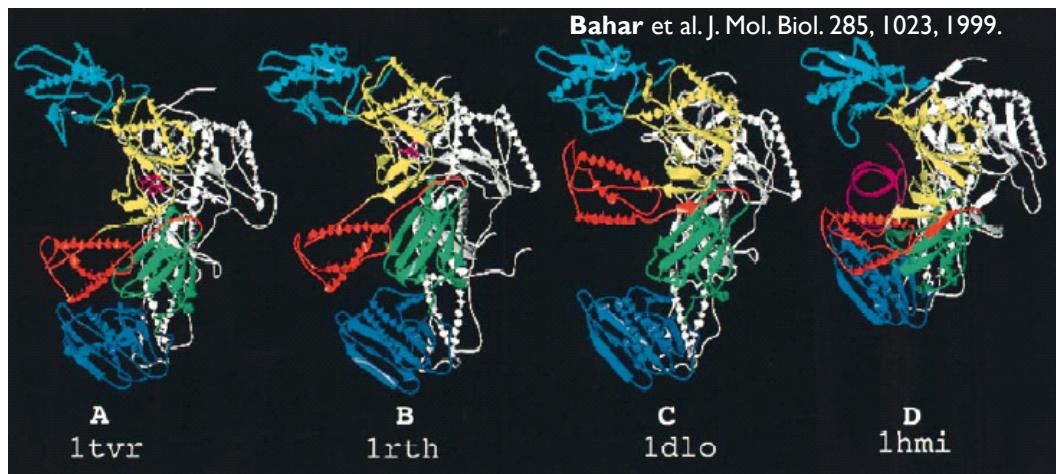
$$\mathbf{d} = [\Delta x_1 \ \Delta y_1 \ \Delta z_1 \dots \ \Delta z_N]^T$$

See...

Z Yang, P Marek and I Bahar, *PLoS Comp Biology* 2009

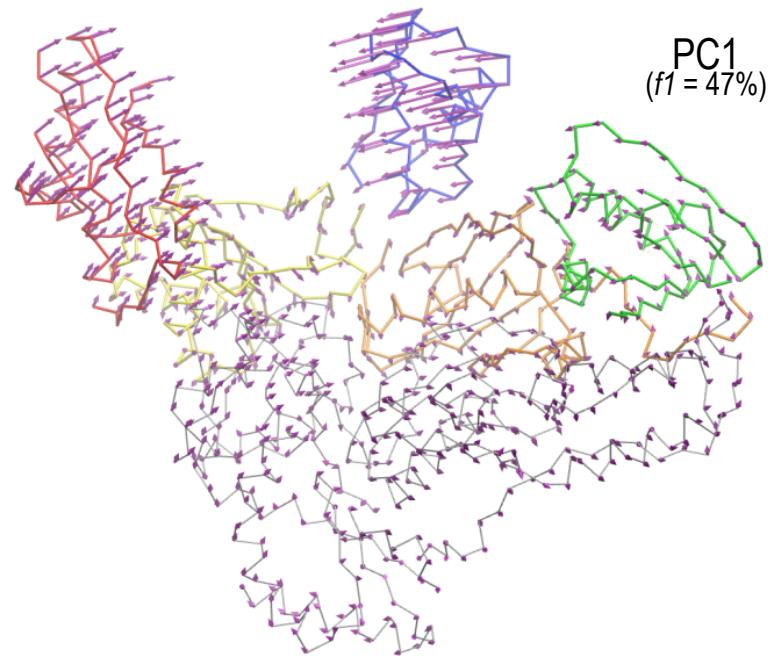
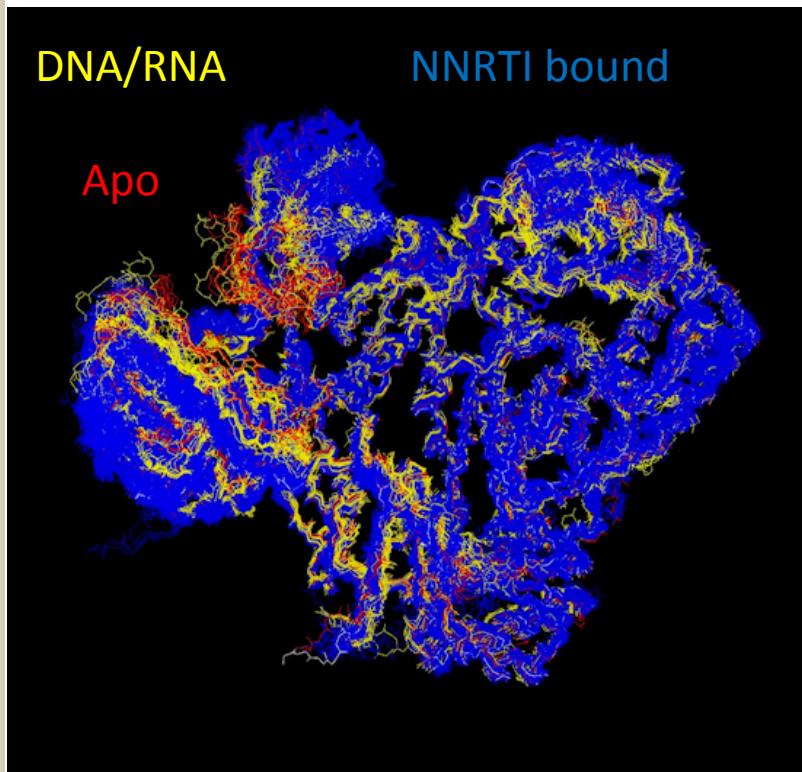
# Dynamics inferred from known structures

Comparison of static structures available in the PDB for the same protein in different form has been widely used is an **indirect** method of inferring dynamics.

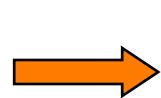


Different structures resolved for HIV-1 reverse transcriptase (RT)

# Principal Component Analysis (PCA)

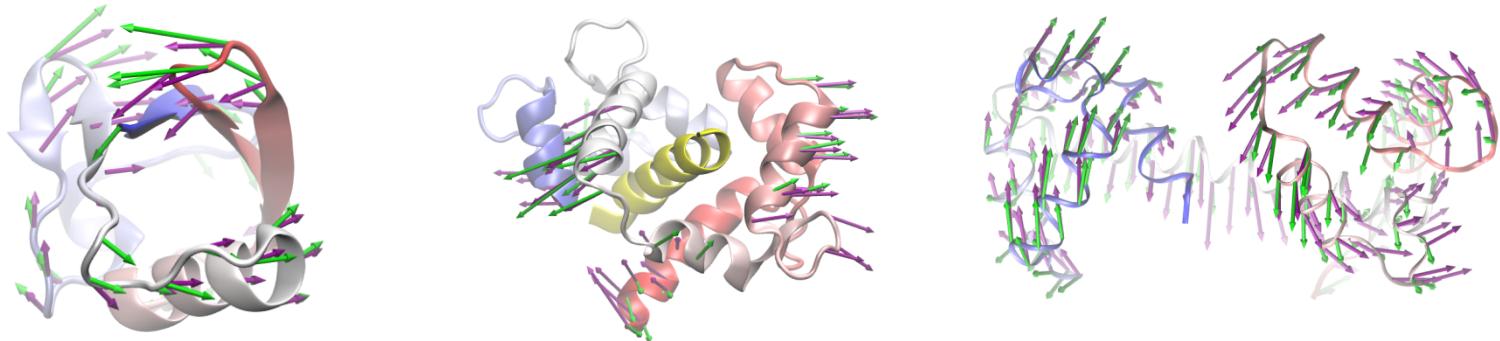


$$\mathbf{C}^{(ij)} = \begin{bmatrix} \langle \Delta x_i \Delta x_j \rangle & \langle \Delta x_i \Delta y_j \rangle & \langle \Delta x_i \Delta z_j \rangle \\ \langle \Delta y_i \Delta x_j \rangle & \langle \Delta y_i \Delta y_j \rangle & \langle \Delta y_i \Delta z_j \rangle \\ \langle \Delta z_i \Delta x_j \rangle & \langle \Delta z_i \Delta y_j \rangle & \langle \Delta z_i \Delta z_j \rangle \end{bmatrix}$$



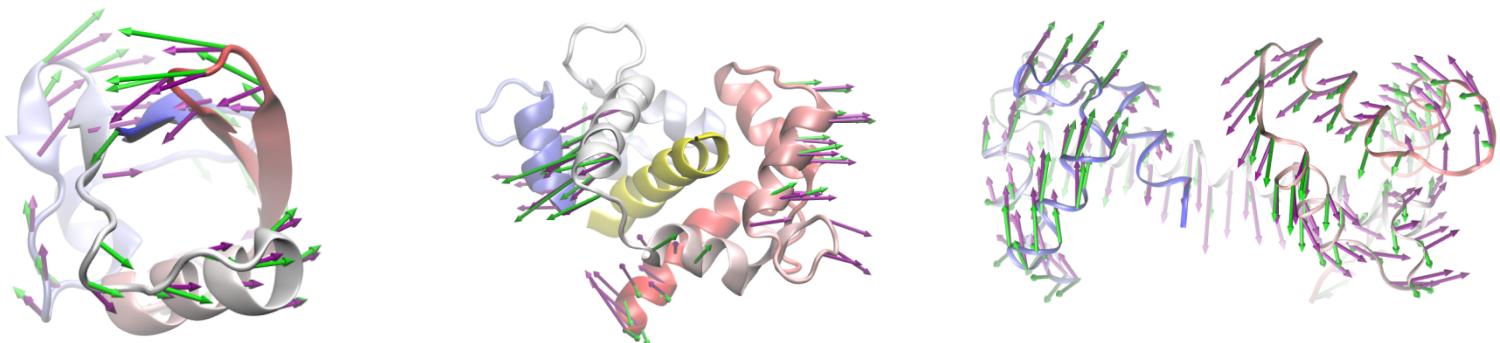
$$\mathbf{C} = \mathbf{P} \mathbf{S} \mathbf{P}^T = \sum_{i=1}^{3N} \sigma_i \mathbf{p}^i \mathbf{p}^{i\top}$$

# Global motions inferred from theory and experiments



- PCA of the ensemble of resolved structures
- ANM analysis of a single structure from the ensemble

# Global motions inferred from theory and experiments



The intrinsic dynamics of enzymes plays a dominant role in determining the structural changes induced upon inhibitor binding

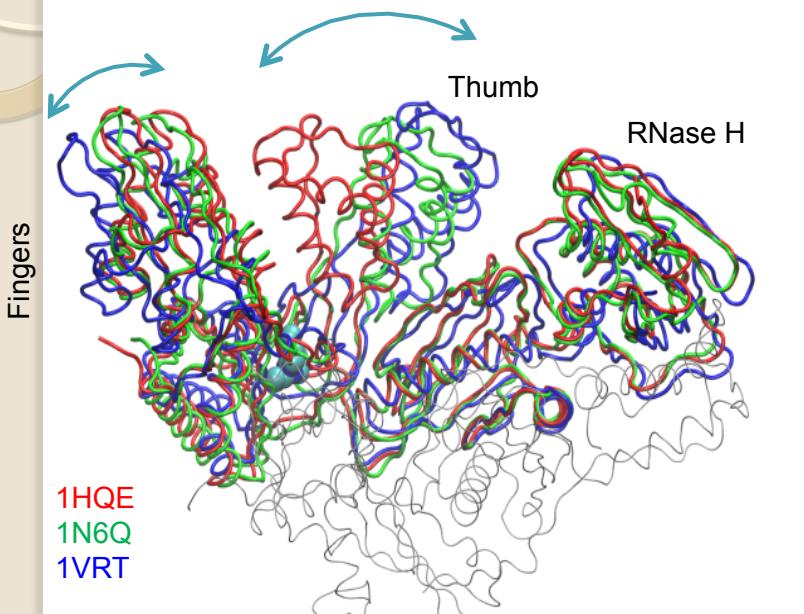
Ahmet Bakan and Ivet Bahar<sup>1</sup>

Department of Computational Biology, School of Medicine, University of Pittsburgh, 3064 BST3, 3501 Fifth Avenue, Pittsburgh, PA 15213

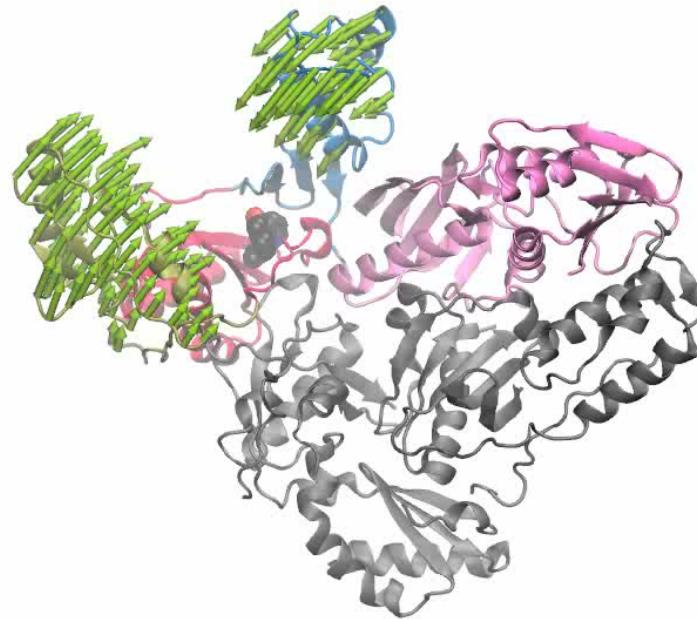
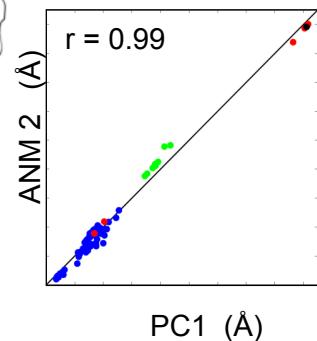
Reference:

Bakan & Bahar (2009) PNAS 106, 14349-54

# Induced Dynamics or Intrinsic Dynamics?



Experiments



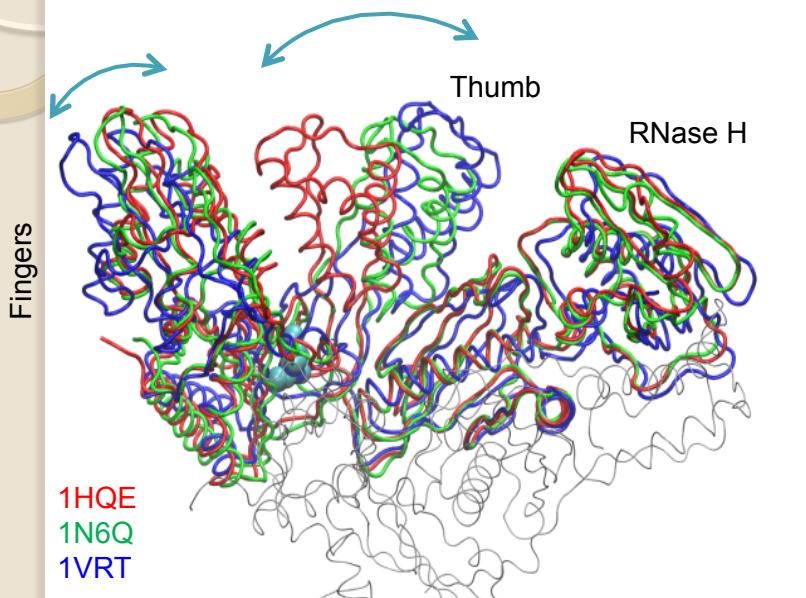
Theory

<http://www.youtube.com/watch?v=1OUzdzm68YY>

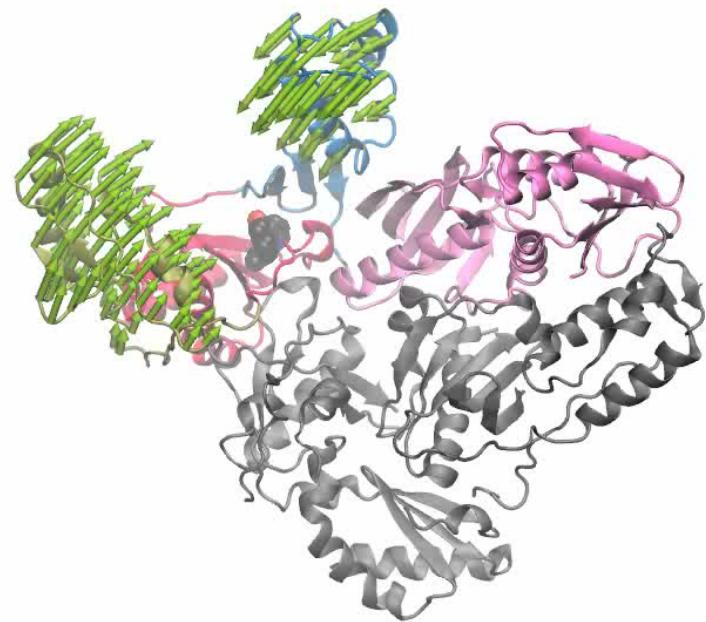
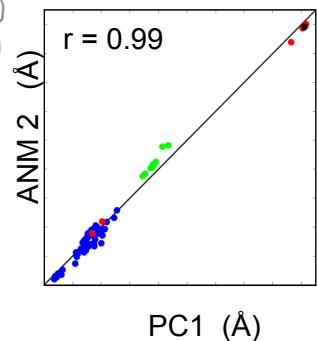
References:

Bakan & Bahar (2009) PNAS 106, 14349-54.

# Soft modes enable **functional** movements



Experiments



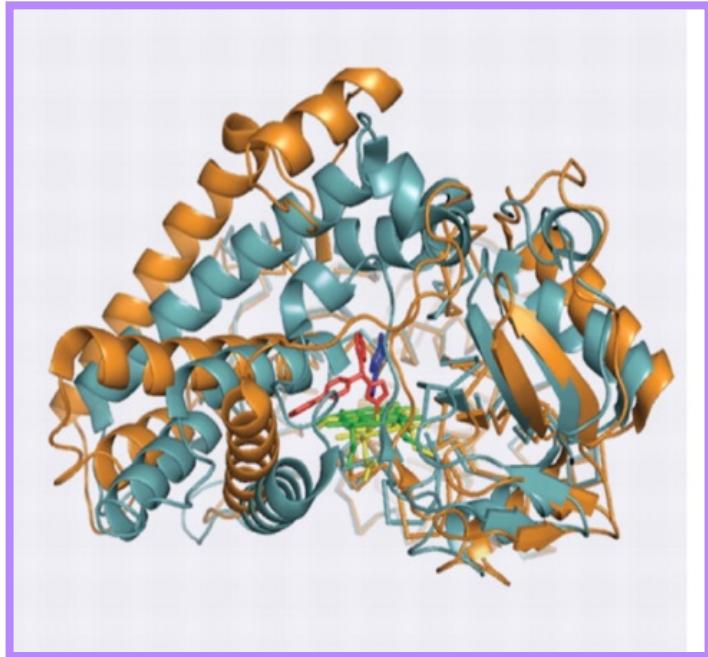
Theory

<http://www.youtube.com/watch?v=1OUzdzm68YY>

References:

Bakan & Bahar (2009) PNAS **106**, 14349-54.

# Intrinsically accessible motions enable Optimal binding of substrate or drugs



Conformational flexibility +  
sequence variability mediates  
**substrate selectivity**

- Two conformations of P450-CYP2B4:  
**open** (orange) with a large substrate (bifonazole, red), and  
**closed** (light blue) with the smaller substrate 4-(4-chlorophenyl) imidazole (blue)

See...

N. Tokuriki and D. S. Tawfik (2009) *Science* **324**: 203-207



# ProDy for exploring conformational space

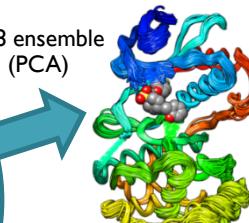
Protein Dynamics Analysis in Python

User inputs a protein sequence

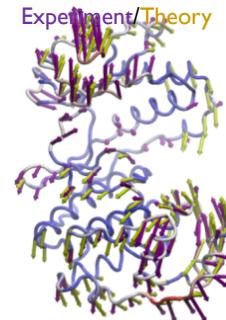
>IA9U:A|PDBID|CHAIN  
GSSHHHHH-HSSGLVPRGSHMSQERP  
TFYRQELNKTIVWEVPERYQNLSPVG  
SGAYGSVCAAFDTKTRGLRAVKLKS  
RPFQSIIHAKRTYRELRLKKHMKHEN  
VIGLLDVFT.....

ProDy identifies, retrieves, aligns, and analyzes (PCA) structures that match the input sequence

p38 ensemble (PCA)



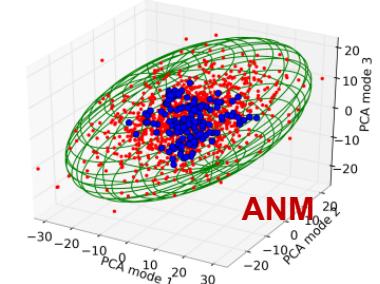
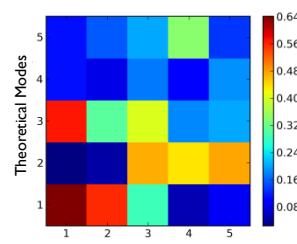
Experiment/Theory



p38 network model (ANM)

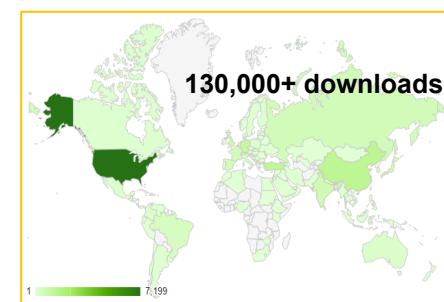
User can compare experimental and theoretical models

Overlap table

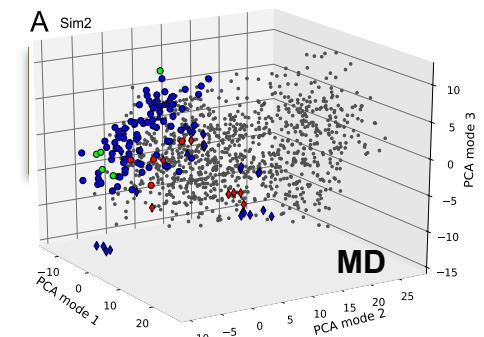


## Growth of Source Code and Usage

	Releases	Downloads	Visits <sup>2</sup>	Unique <sup>3</sup>
Nov '10 - Oct '11	19	8,530	8,678	2,946
Nov '11 - Oct '12	15	35,108	16,472	6,414
Nov '12 - Oct '13	8*	87,909	19,888	8,145
<b>Total</b>	<b>42</b>	<b>131,547</b>	<b>45,038</b>	<b>17,505</b>



Source <http://www.google.com/analytics/>



Bakan & Bahar, PSB 2011, 181-192

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