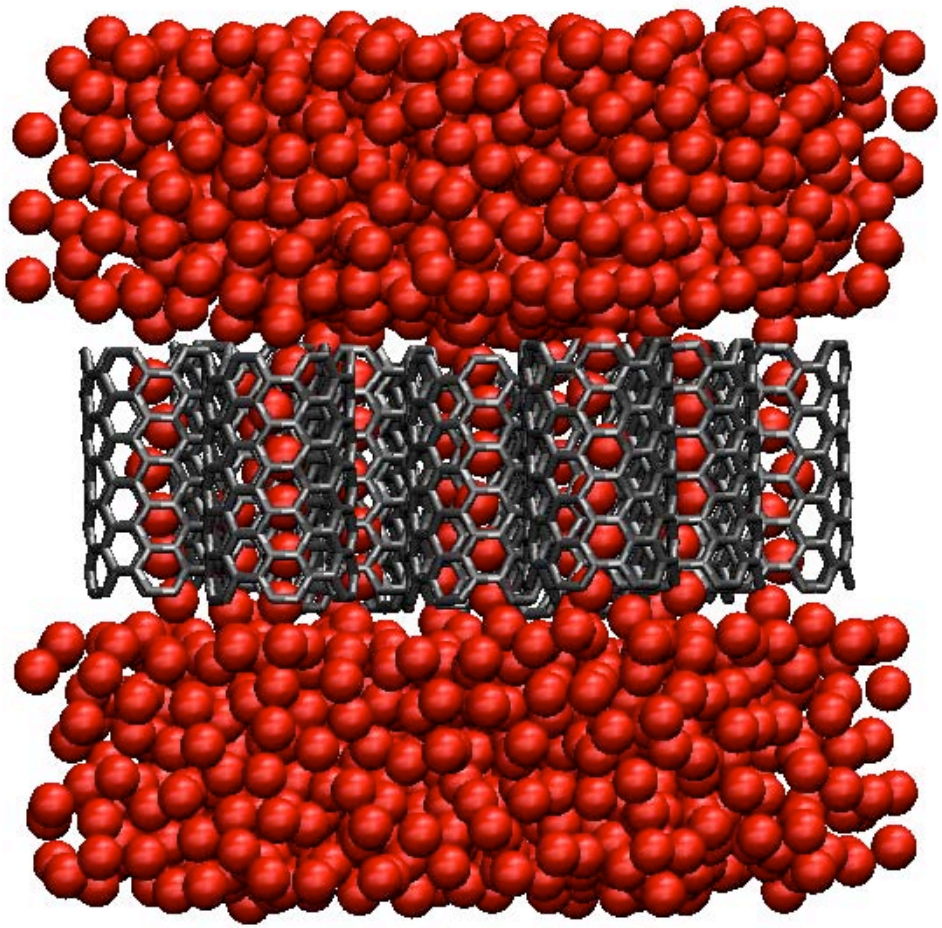
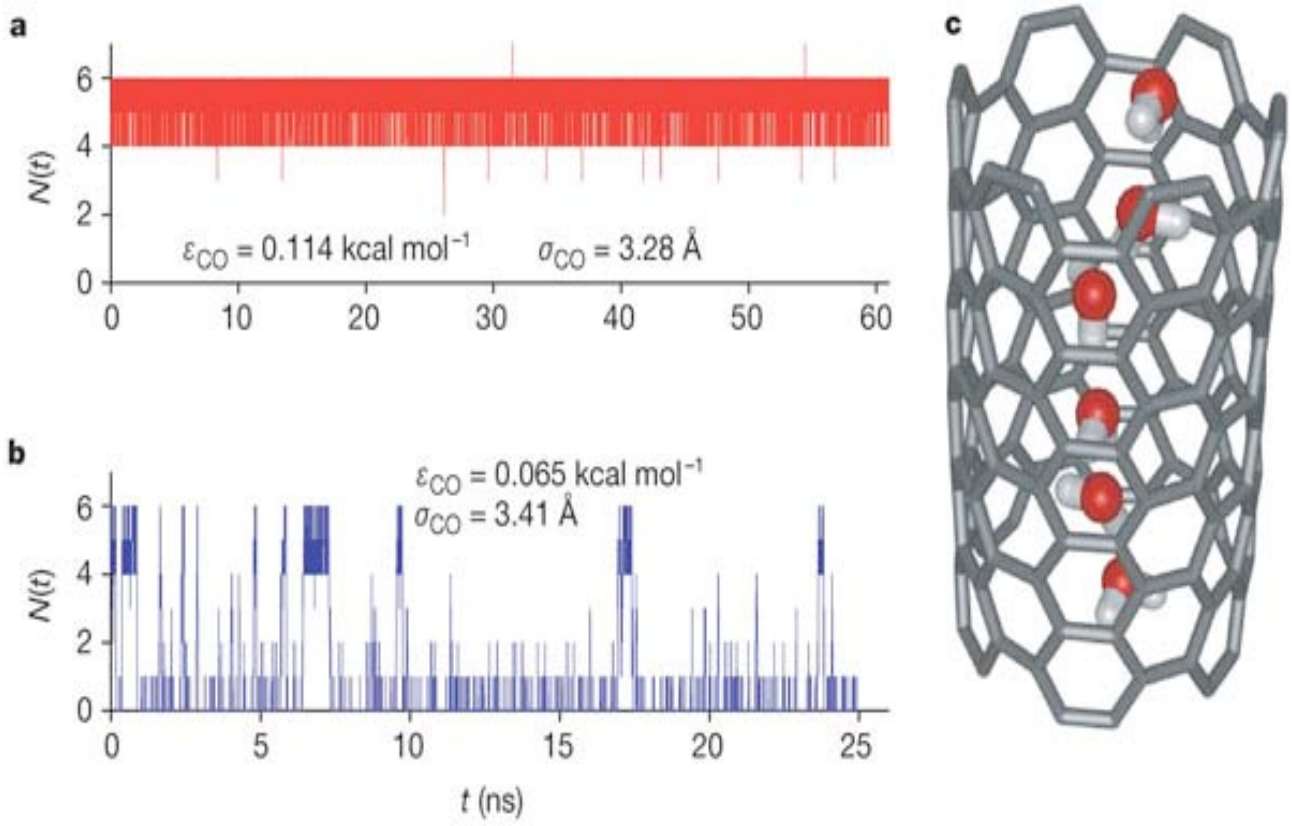
Carbon Nanotubes
Hydrophobic channels - Perfect Models for Membrane Water Channels



A balance between the size and hydrophobicity

Water-nanotube interaction can be easily modified



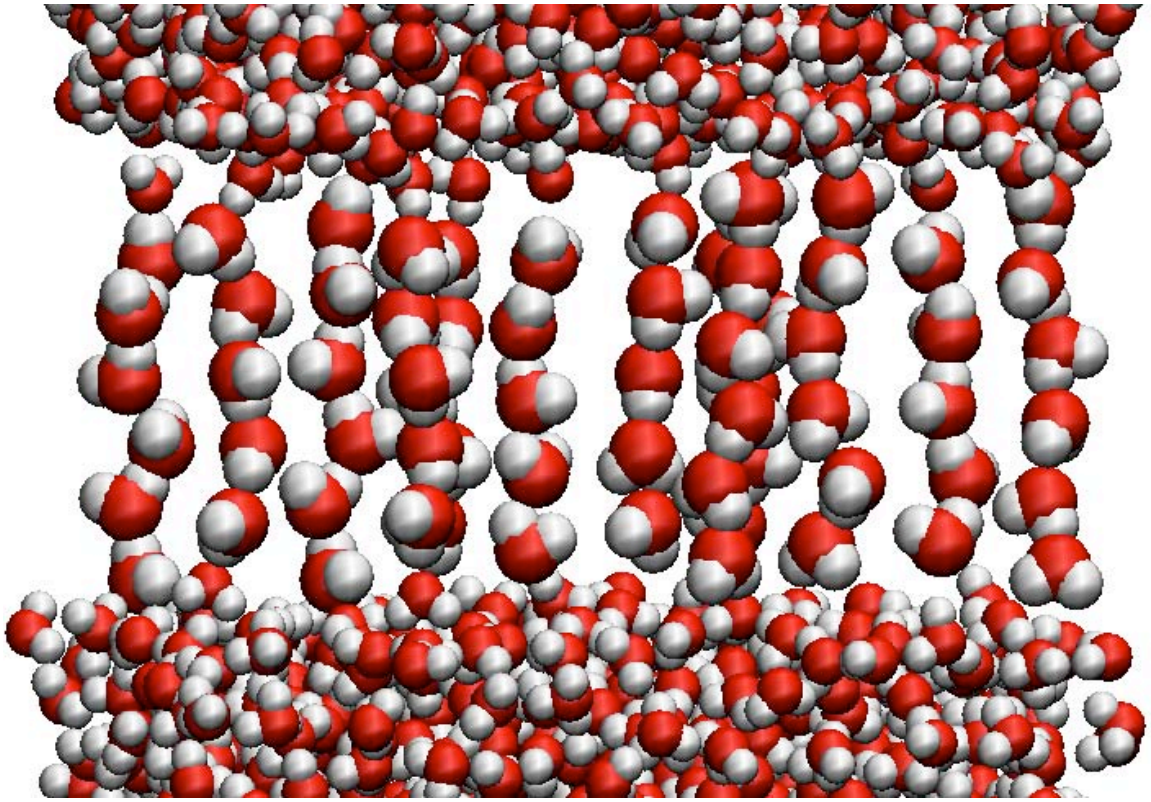
Modifying charges
Modifying vdW parameters

Hummer, et. al., Nature, 414: 188-190, 2001

Carbon Nanotubes
Hydrophobic channels - Perfect Models for Membrane Water Channels

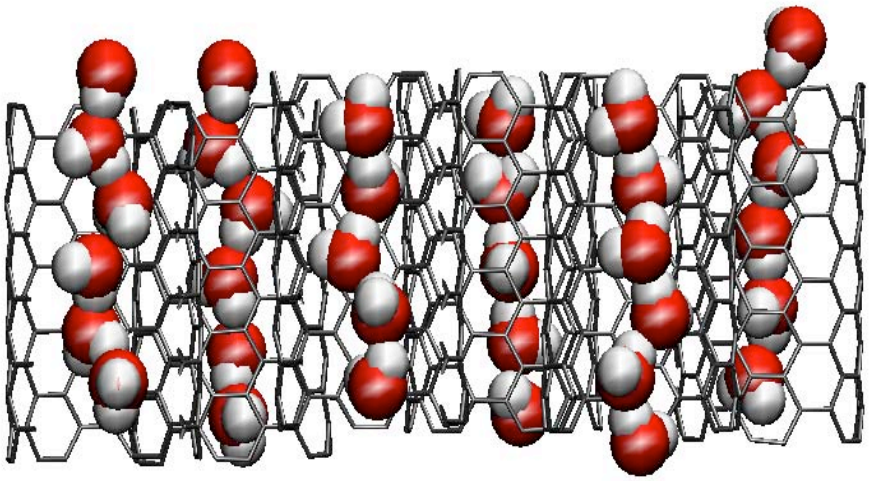
- Much better statistics
- No need for membrane and lipid molecules

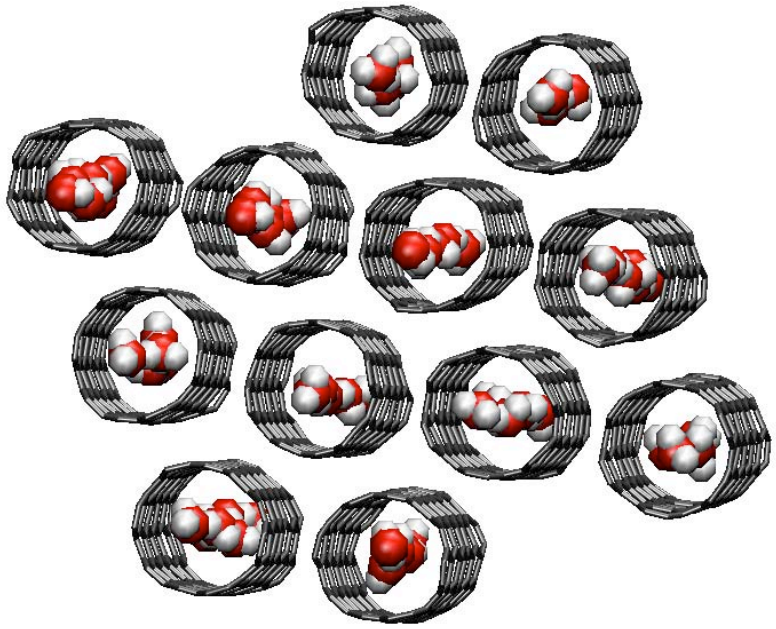
Carbon Nanotubes
Hydrophobic channels - Perfect Models for Membrane Water Channels



- Much better statistics
- No need for membrane and lipid molecules

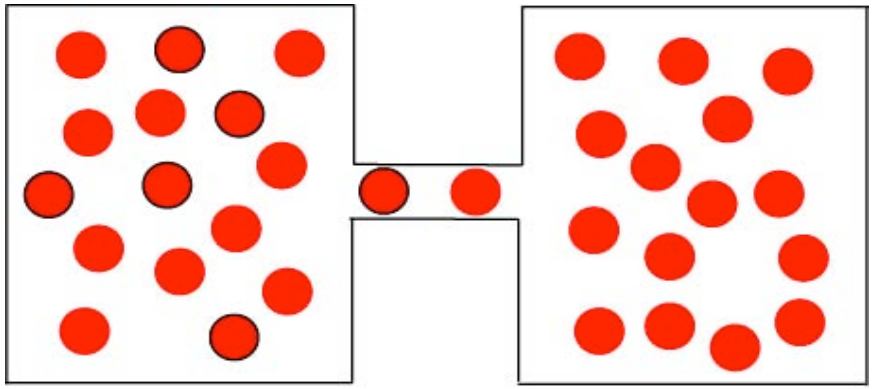
Water Single-files in Carbon Nanotubes





Water files form polarized chains in nanotubes

Calculation of Diffusion Permeability from MD



Φ_0: number of water molecules crossing the channel from the left to the right in unit time

$$p_d = \frac{V_w}{N_A}\Phi_0$$

Φ_0 can be directly obtained through equilibrium MD simulation by counting "full permeation events"

Chemical Potential of Water

$$\mu_w = \mu_w^o + RT \ln X_w + PV_w$$

μ_w^o : **standard chemical potential of water**

X_w : **molar fraction of water**

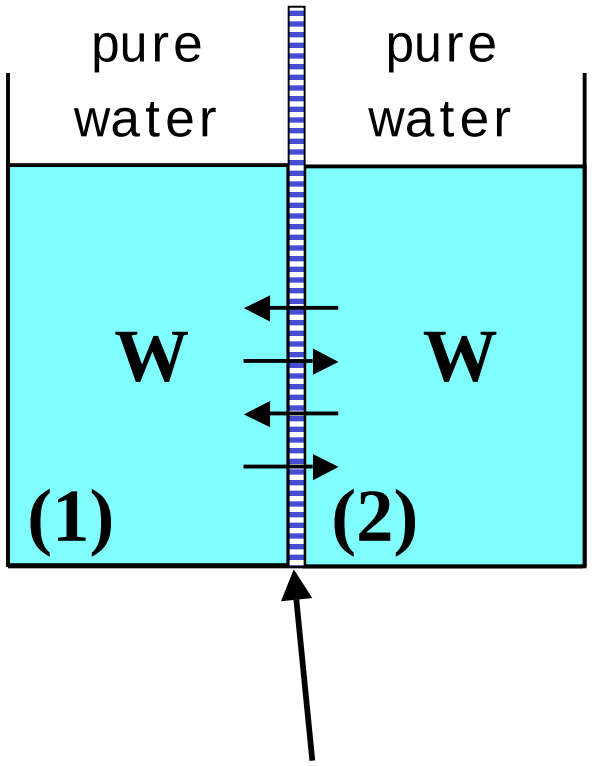
R : **the gas constant**

T : **temperature**

P : **pressure**

V_w : **molar volume of water**

$X_w = 1 \Rightarrow \ln X_w = 0$



Water flow in either direction is the same, i.e., no net flow of water.

Solutes Decrease the Chemical Potential of Water

$$\mu_w = \mu_w^o + RT \ln X_w + PV_w$$

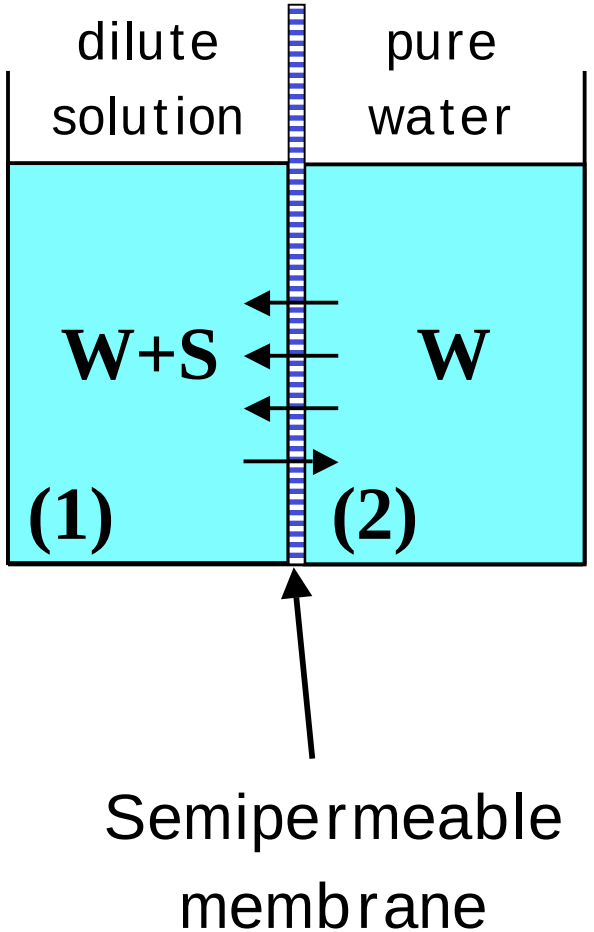
Addition of an impermeable solute to one compartment drives the system out of equilibrium.

$$RT \ln X_w(1) < RT \ln X_w(2)$$

$$\Rightarrow \mu_w(1) < \mu_w(2)$$

Water establishes a net flow from compartment (2) to compartment (1).

$X_w(1) < 1 \quad X_w(2) = 1$



Establishment of Osmotic Equilibrium

$$@\text{equilibrium}: \mu_w(1) = \mu_w(2)$$

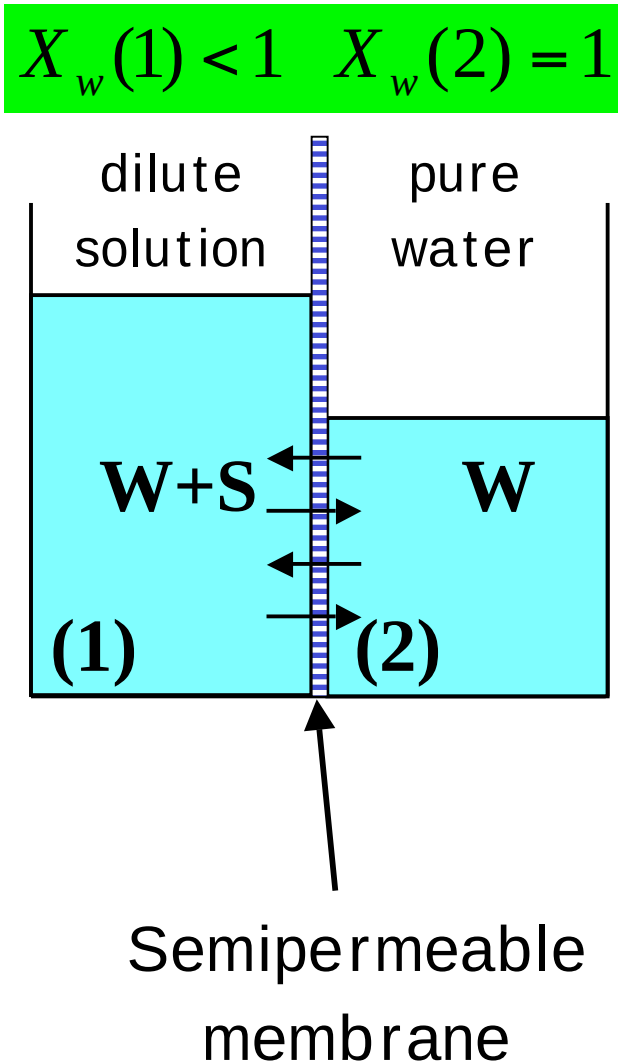
At equilibrium, the chemical potential of any species is the same at every point in the system to which it has access.

$$\mu_w^o(1) + RT \ln X_w(1) + P(1)V_w =$$

$$\mu_w^o(2) + RT \ln X_w(2) + P(2)V_w$$

$$RT \ln X_w(1) + P(1)V_w = P(2)V_w$$

$$\Delta \boldsymbol{PV_w} = -\boldsymbol{RT} \ln \boldsymbol{X_w}(1)$$



Establishment of an Osmotic Equilibrium

$$\Delta PV_w = -RT \ln X_w \quad (1)$$

Solute molar fraction in physiological (dilute) solutions is much smaller than water molar fraction.

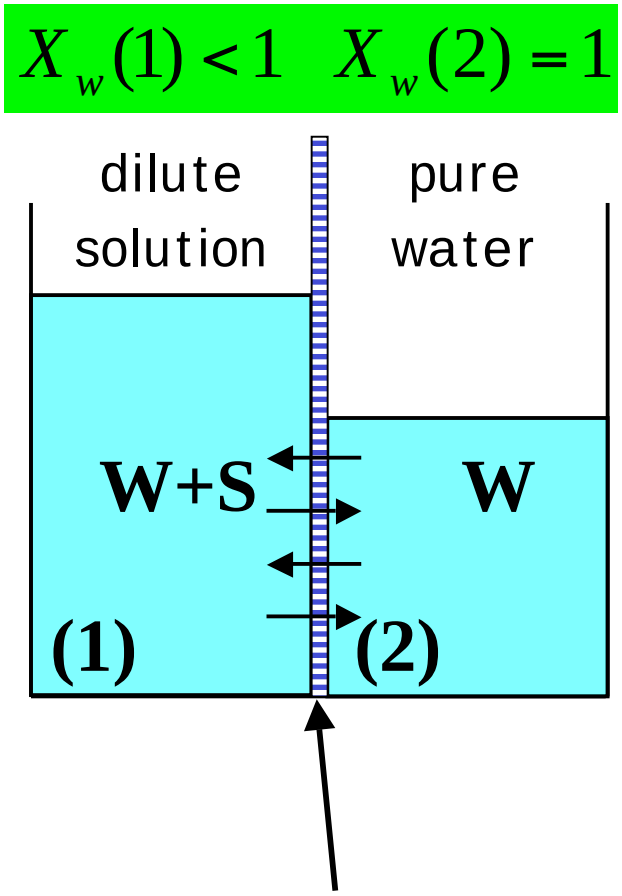
$$X_w + X_s = 1 \; ; \; X_s << 1$$

$$\Rightarrow \ln X_w = \ln(1 - X_s) \cong -X_s$$

$$\Delta PV_w = RTX_s$$

$$\Rightarrow \prod = \Delta P = \frac{RT}{V_w} X_s$$

Osmotic pressure



Establishment of an Osmotic Equilibrium

$$\prod = \Delta P = \frac{RT}{V_w} X_s$$

Solute concentration (~0.1M) in physiological (dilute) solutions is much smaller than water concentration (55M).

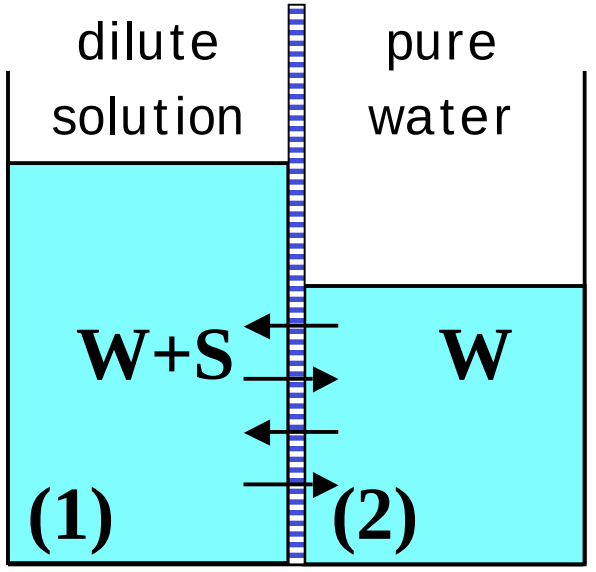
$$X_s = \frac{n_s}{n_s + n_w} \approx \frac{n_s}{n_w} = \frac{n_s}{n_w}\frac{V_w}{V_w}$$

$n_s << n_w$

$$= \frac{n_s}{V_{tot}} V_w = C_s V_w$$

$$\Pi = \Delta P = \frac{RT}{V_w} C_s V_w = RTC_s$$

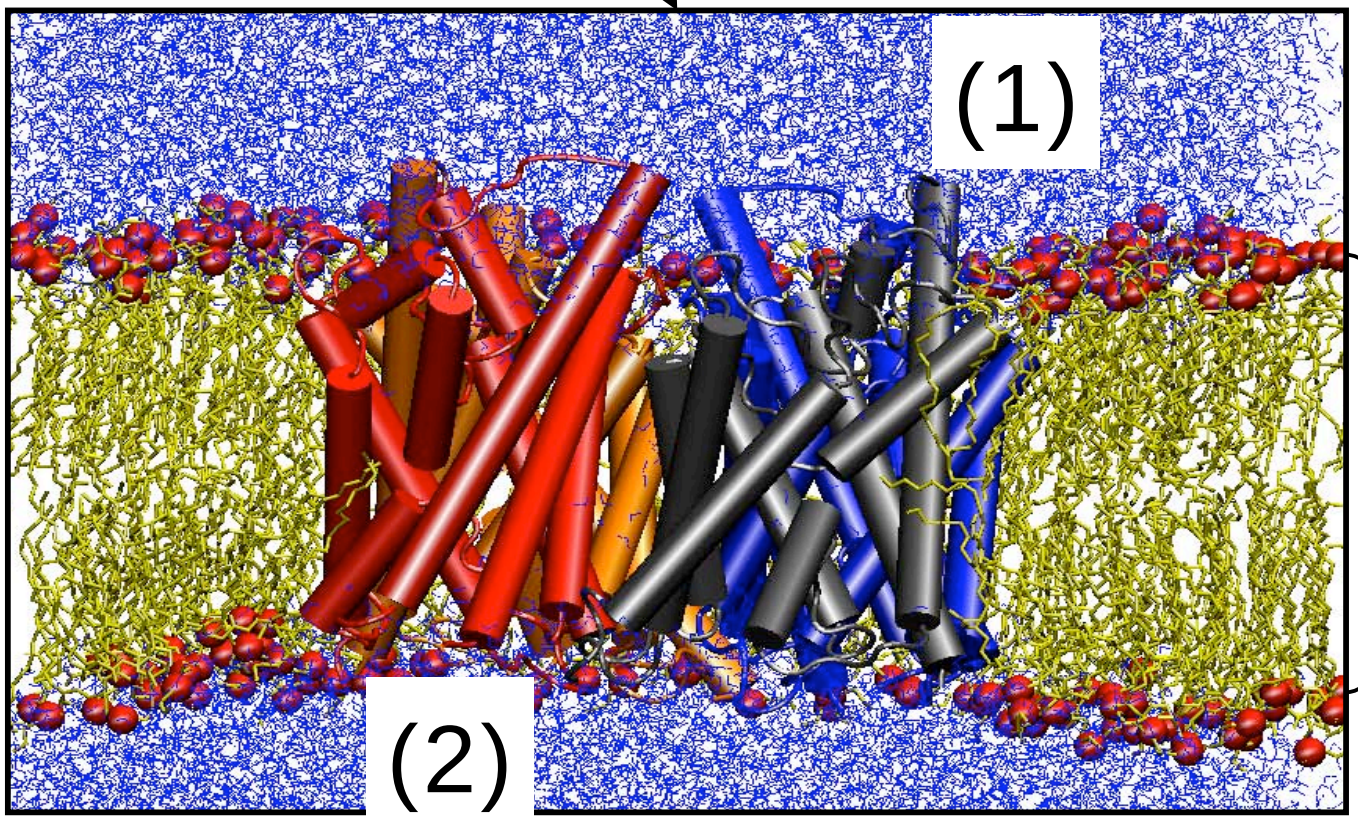
$X_w(1) < 1 \quad X_w(2) = 1$



$$\Delta\Pi = \Delta P = RT\,\Delta C_s$$

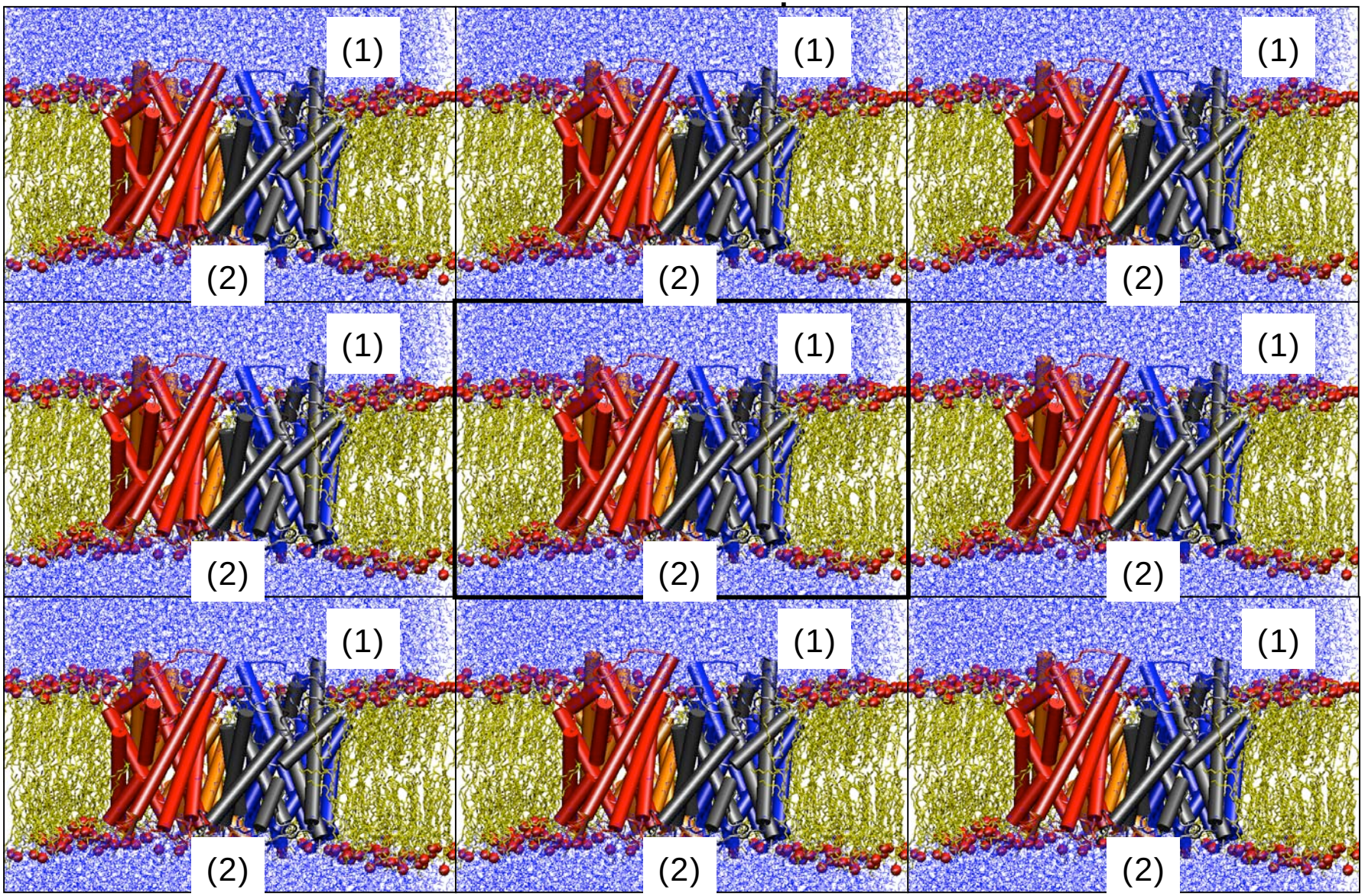
Simulation of osmotic pressure induced water transport may be done by adding salt to one side of the membrane.

$NaCl \rightarrow Na^{+} + Cl^{-}$



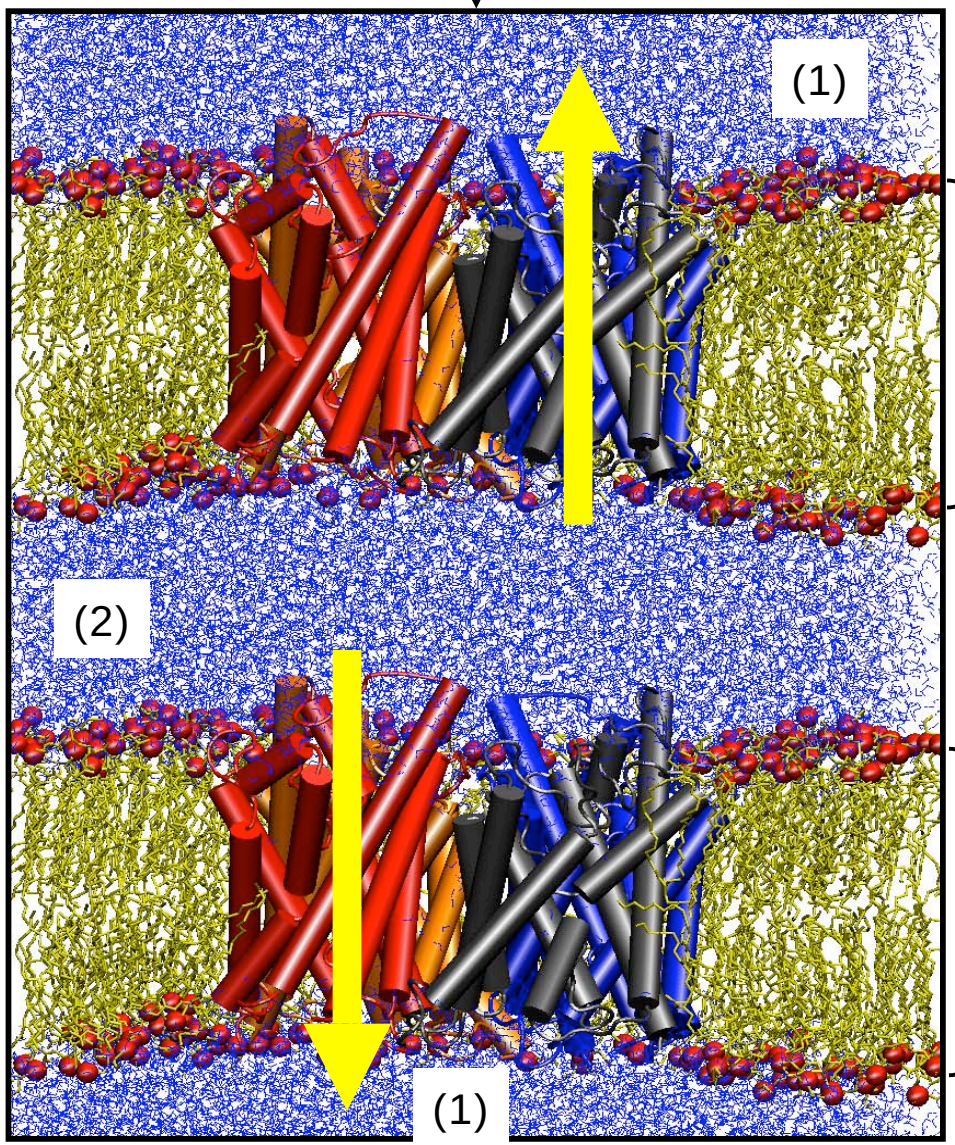
There is a small problem with this setup!

Problem: The solvents on the two sides of a membrane in a conventional periodic system are



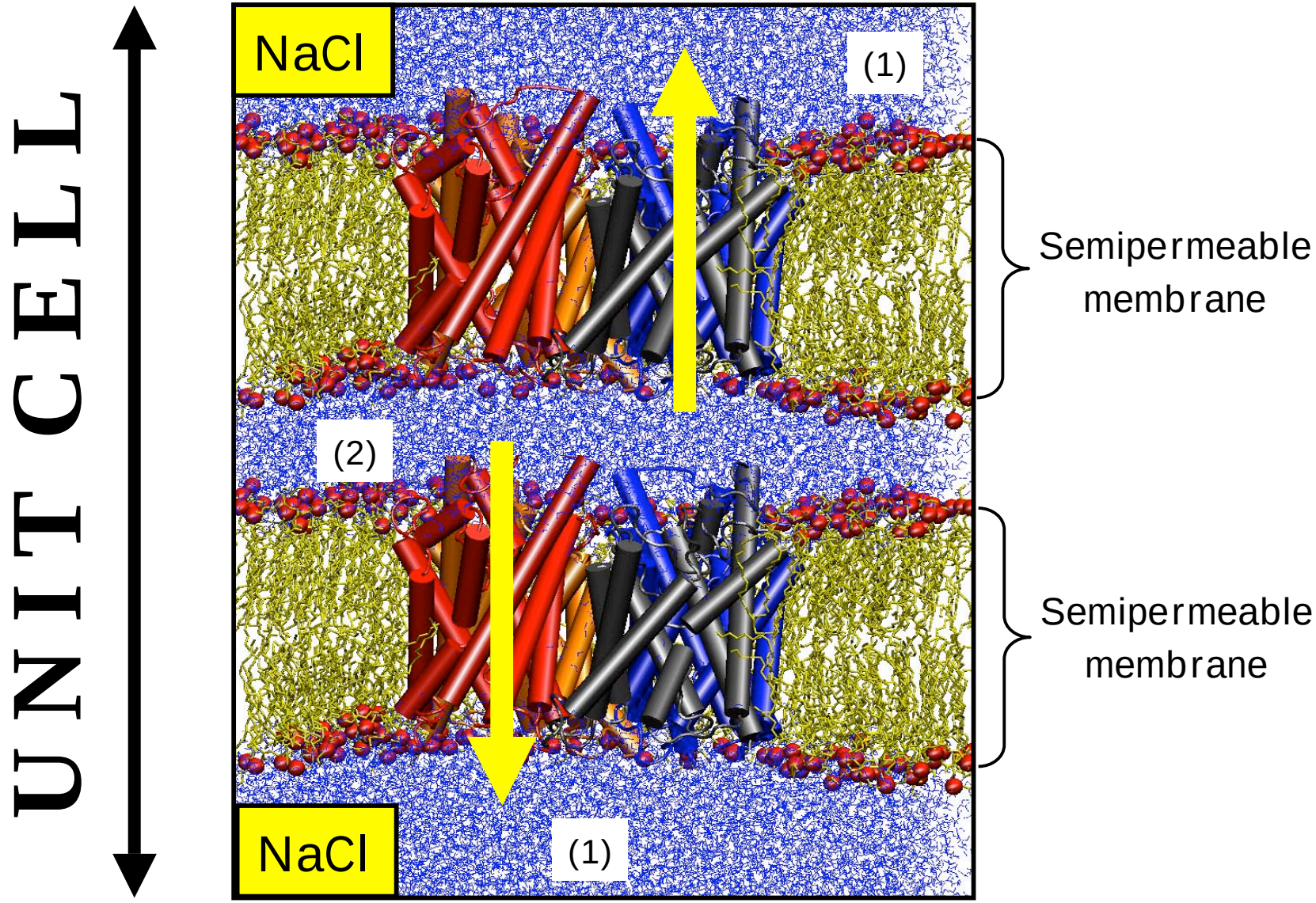
We can include more layers of membrane and water to create two compartment of water that are not in contact

NaCl



Semipermeable membrane

Semipermeable membrane


UNIT CELL
NaCl
(1)
Semipermeable membrane
(2)
Semipermeable membrane
NaCl
(1)

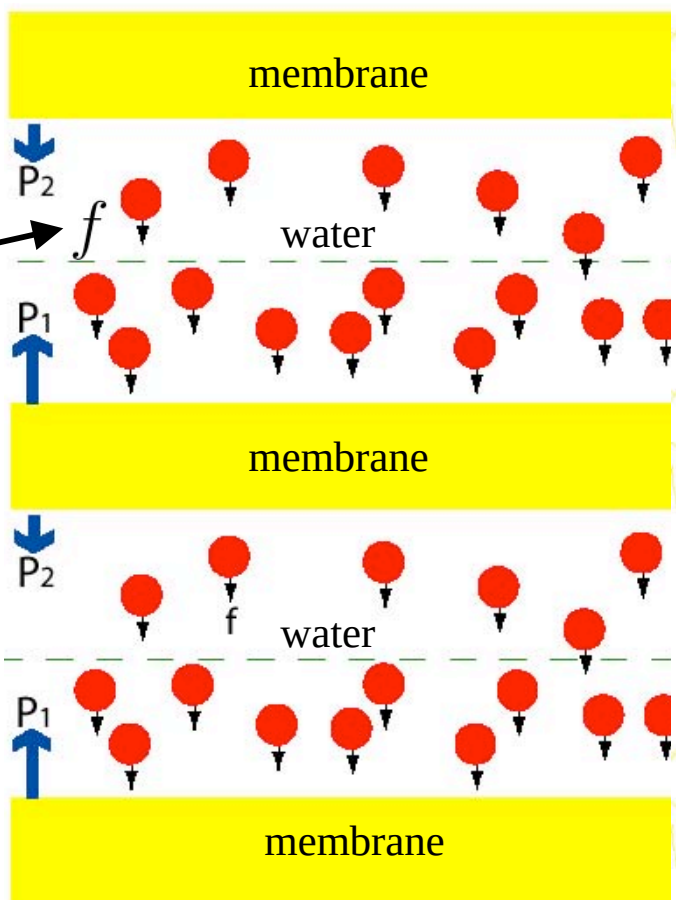
Realizing a Pressure Difference in a Periodic System

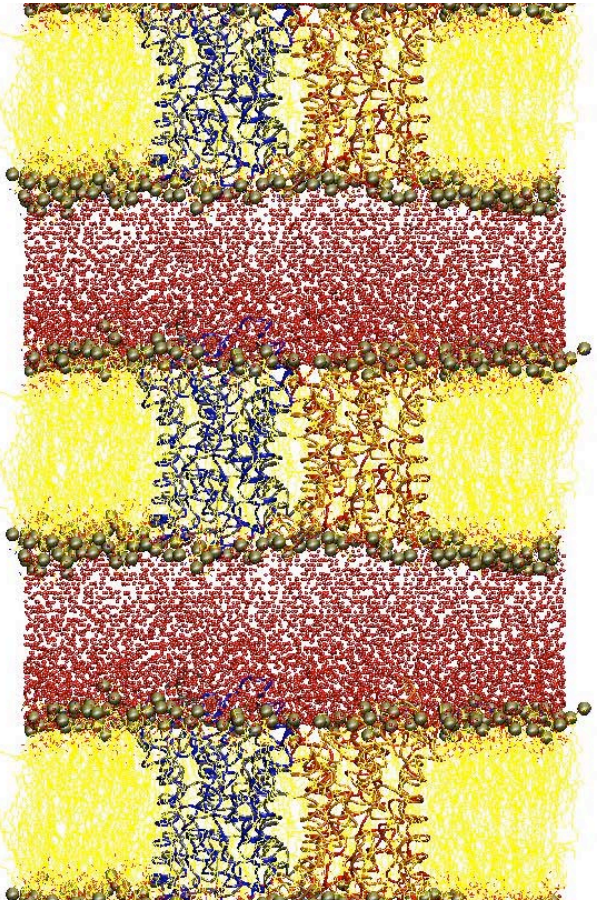
$$P_1 = P_2 + nf \Rightarrow \Delta P = nf / A$$

Fangqiang Zhu

f is the force on each water molecule, for *n* water molecules

The overall translation of the system is prevented by applying constraints or counter forces to the membrane.

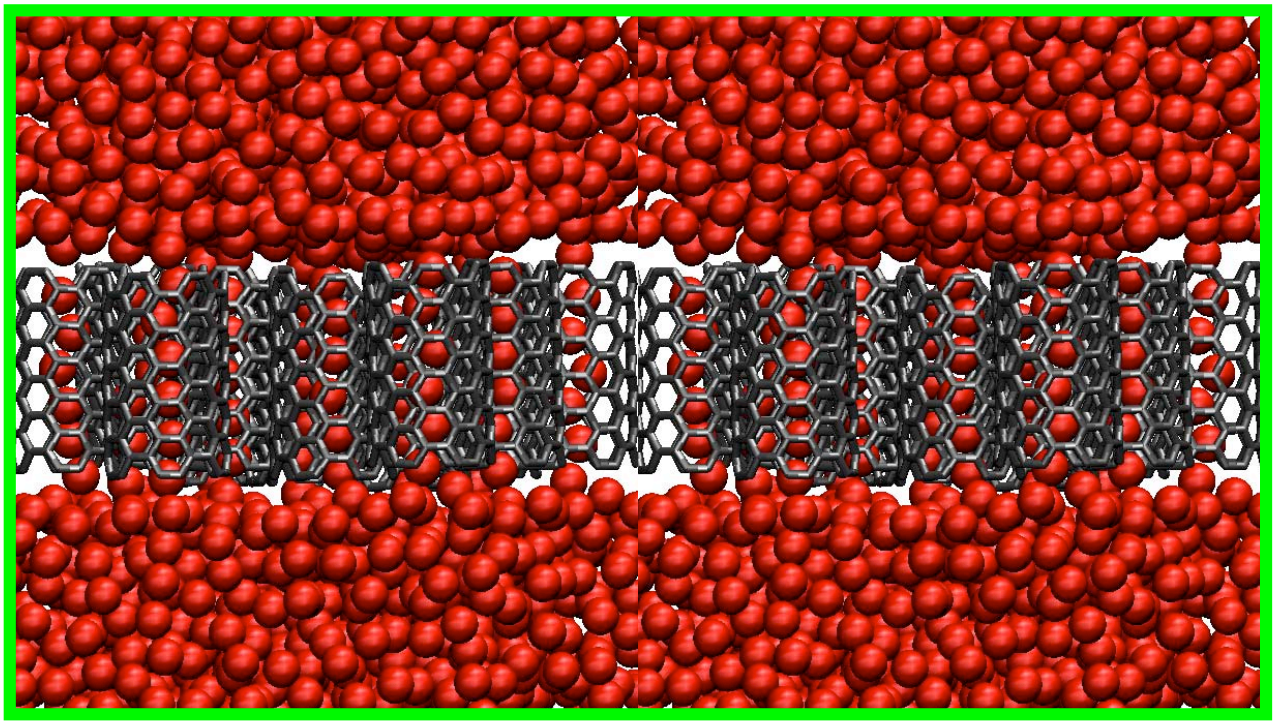




F. Zhu, et al., *Biophys. J.* 83, 154 (2002).

Applying a Pressure Difference Across the Membrane

$$\Delta P = nf / A$$

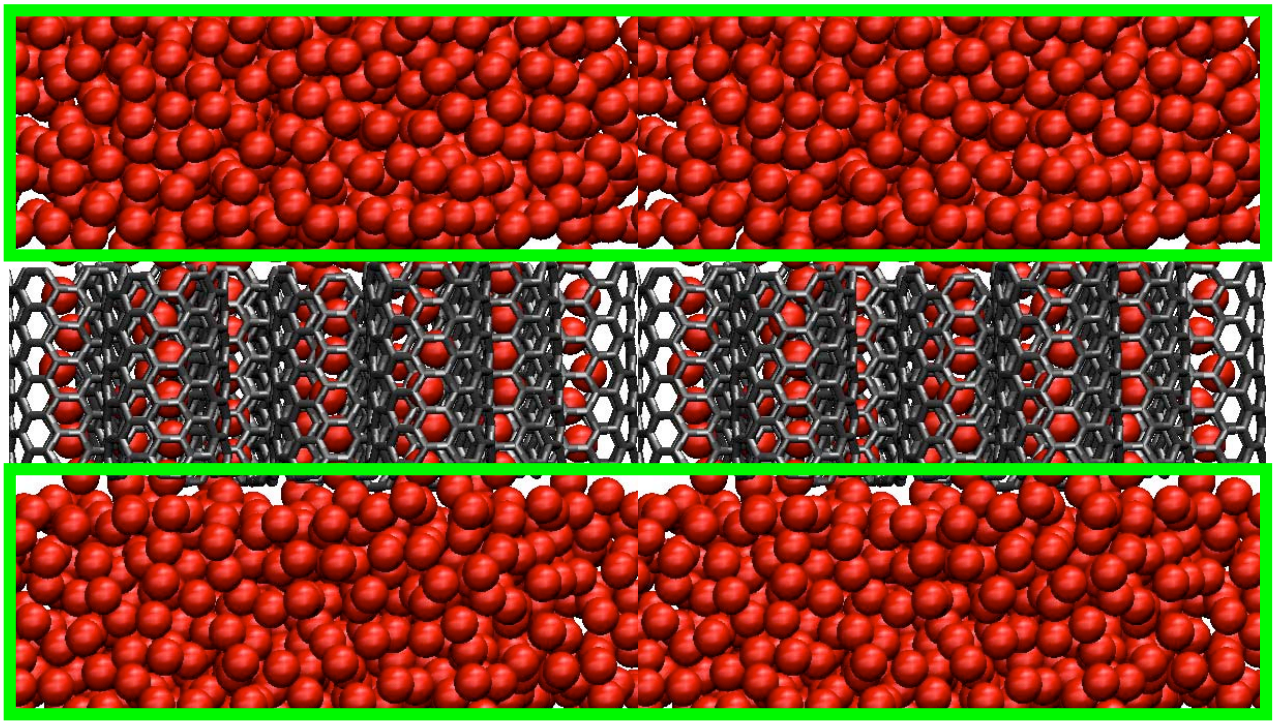


Applying force on all water molecules.

Not a good idea!

Applying a Pressure Difference Across the Membrane

$$\Delta P = nf / A$$

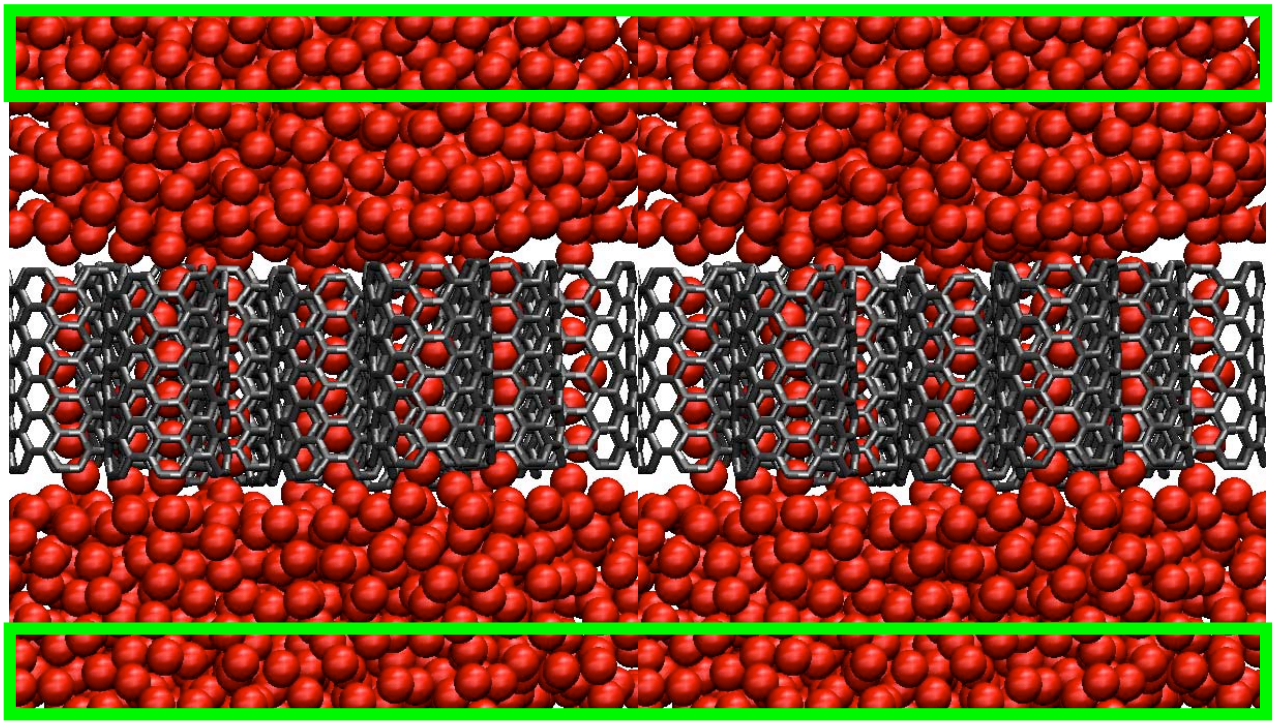


Applying force on bulk water only.

Very good

Applying a Pressure Difference Across the Membrane

$$\Delta P = nf / A$$



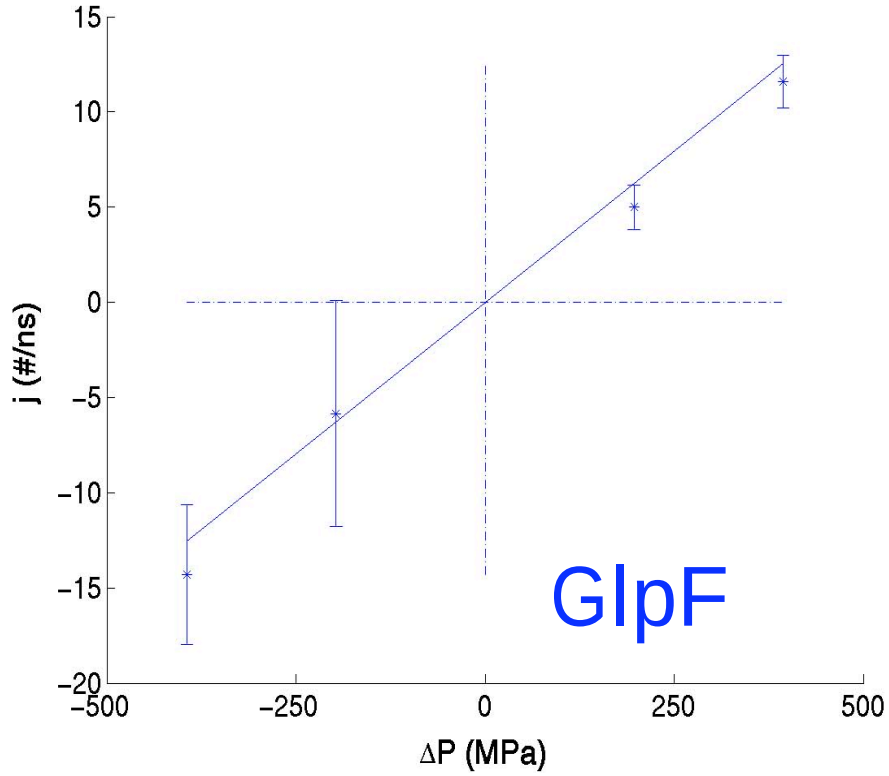
Applying force only on a slab of water in bulk.

Excellent

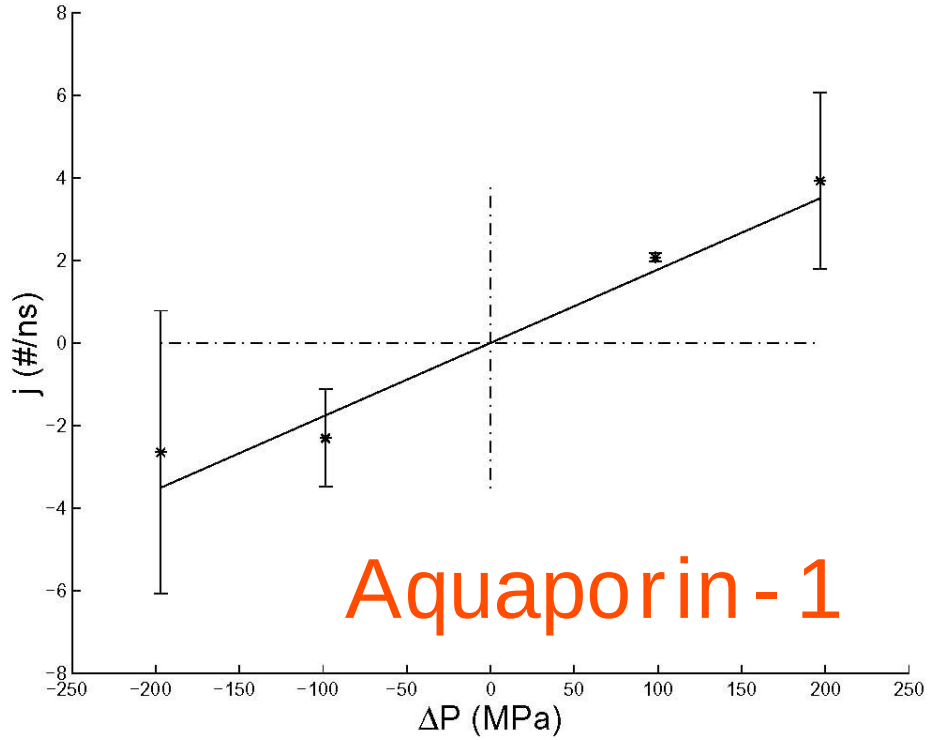
P_f can be calculated from these simulations

$$\Phi_w = P_f A\left(\frac{\Delta P}{RT} - \Delta C_s\right)$$

Calculation of osmotic permeability of water channels



p_f: 1.4×10^{-13} cm^3/s



p_f: 7.0 ± 0.9 × 10^{-14} cm^3/s
Exp: 5.4 – 11.7 × 10^{-14} cm^3/s

Interactive Molecular Dynamics

VMD ◄·····► NAMD