Temperature Echoes in Proteins

- Coherent motion in proteins: Echoes
- Generation of echoes in *ubiquitin* via velocity reassignments
  1) Temperature quench echoes
  2) Constant velocity reassignment echoes
  3) Velocity reassignment echoes

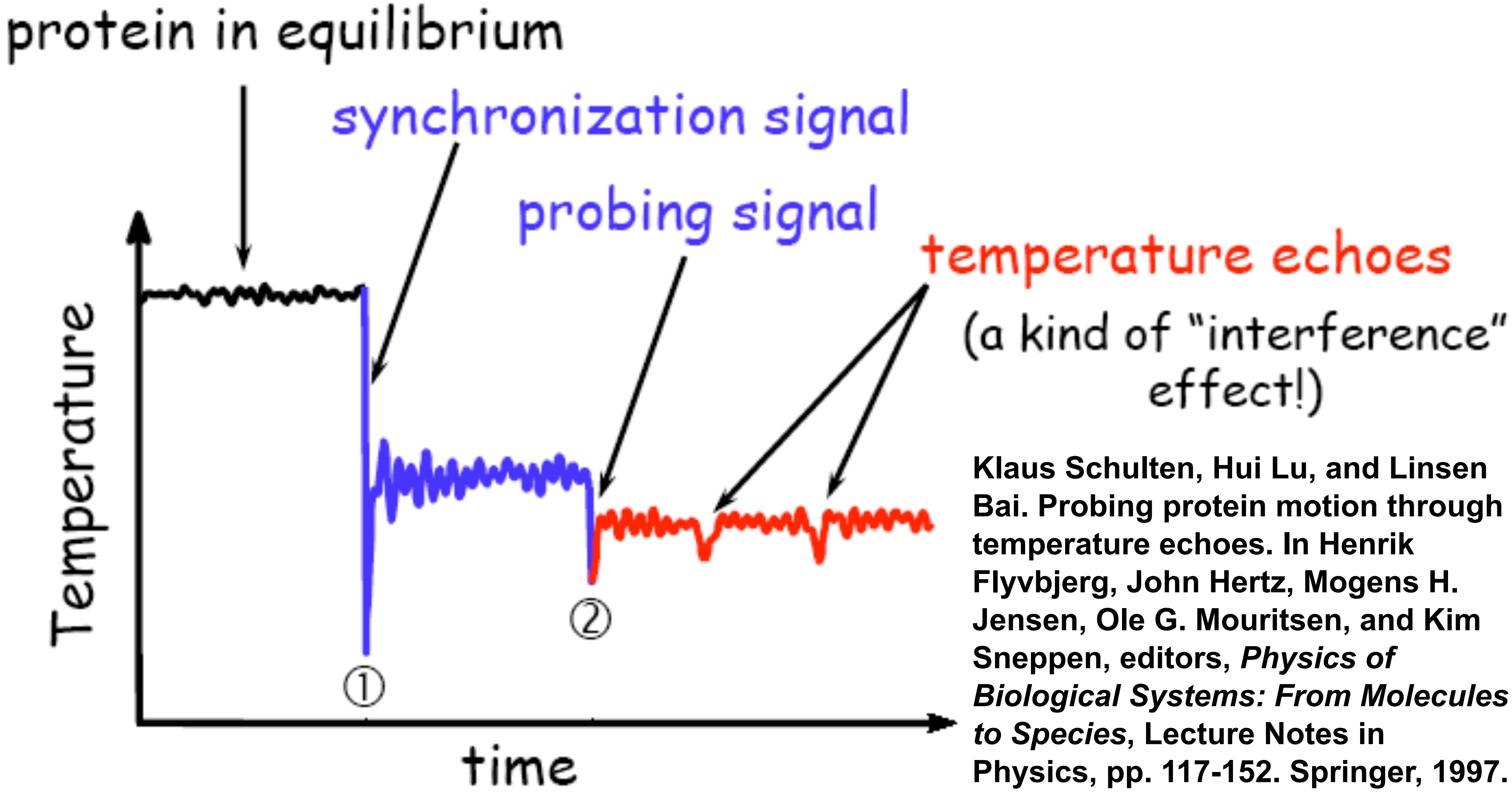
temperature $\Leftrightarrow$ velocities

kinetic temperature:

$$T(t) = \frac{2}{(3N-6)k_B} \sum_{n=1}^{3N-6} \frac{m_n v_n^2(t)}{2}$$

Temperature Echoes

- are sharp, resonance-like features in the time evolution of the protein's temperature
- can be produced through 2 consecutive velocity reassignments



Klaus Schulten, Hui Lu, and Linsen Bai. Probing protein motion through temperature echoes. In Henrik Flyvbjerg, John Hertz, Mogens H. Jensen, Ole G. Mouritsen, and Kim Sneppen, editors, *Physics of Biological Systems: From Molecules to Species*, Lecture Notes in Physics, pp. 117-152. Springer, 1997.

Velocity Reassignments

- protein ≈ collection of weakly interacting harmonic oscillators having different frequencies
- at $t_1=0$ the 1st velocity reassignment: $v_i(0)=\lambda_1 u_i$ synchronizes the oscillators (i.e., make them oscillate in phase)
- at $t_2=\tau$ (delay time) the 2nd velocity reassignment: $v_i(\tau)=\lambda_2 u_i$ probes the degree of coherence of the system at that moment
- degree of coherence is characterized by:

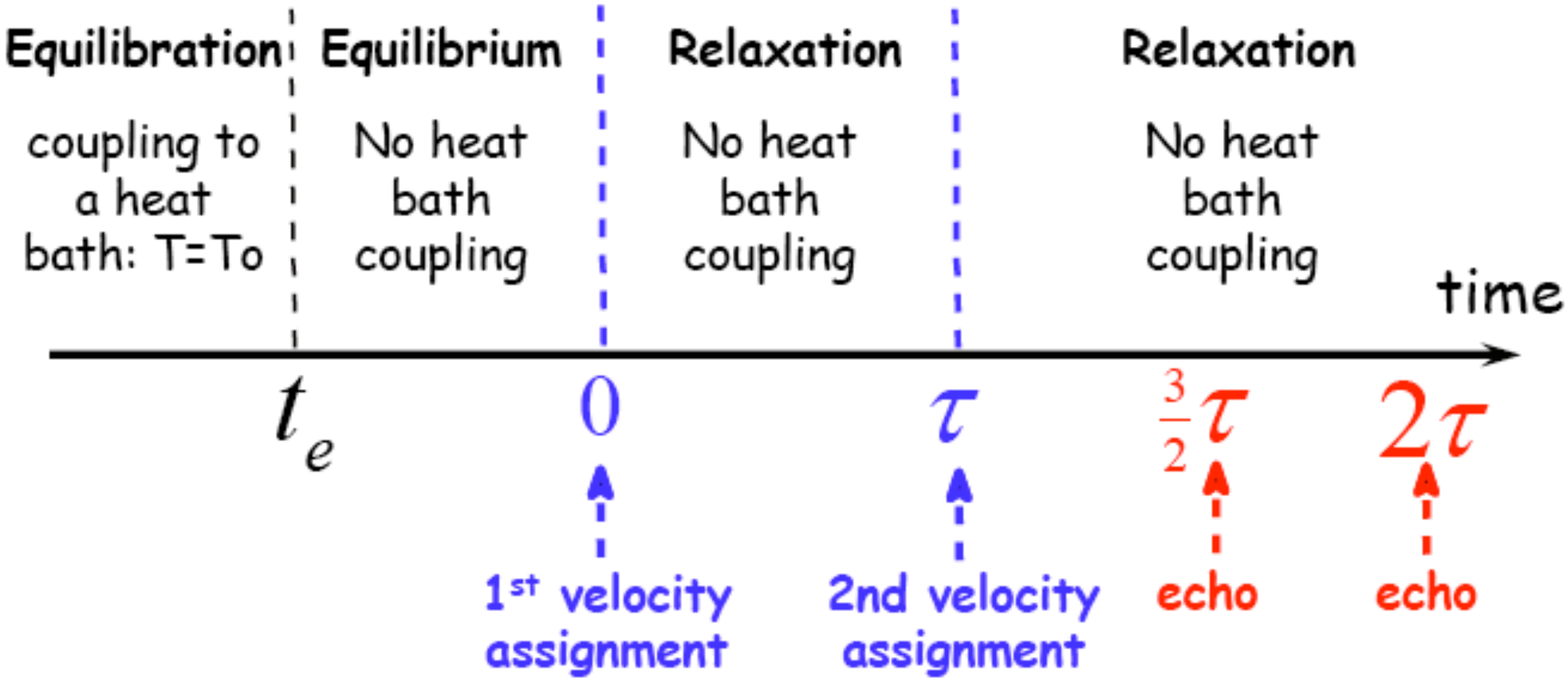
  - the time(s) of the echo(es)
  - the depth of the echo(es)

$$\lambda_1 = \lambda_2 = 0 \quad \Rightarrow \quad \text{temperature quench}$$

$$\lambda_1 = \lambda_2 = 1 \quad \Rightarrow \quad \text{constant velocity reassignment}$$

$$\lambda_1 \neq \lambda_2 \neq 1 \quad \Rightarrow \quad \text{velocity reassignment}$$

Producing Temperature Echoes by Velocity Reassignments in Proteins



Temperature quench echoes: $v_i(0) = v_i(\tau) = 0$

Const velocity reassignment echoes: $v_i(0) = v_i(\tau) = u_i$

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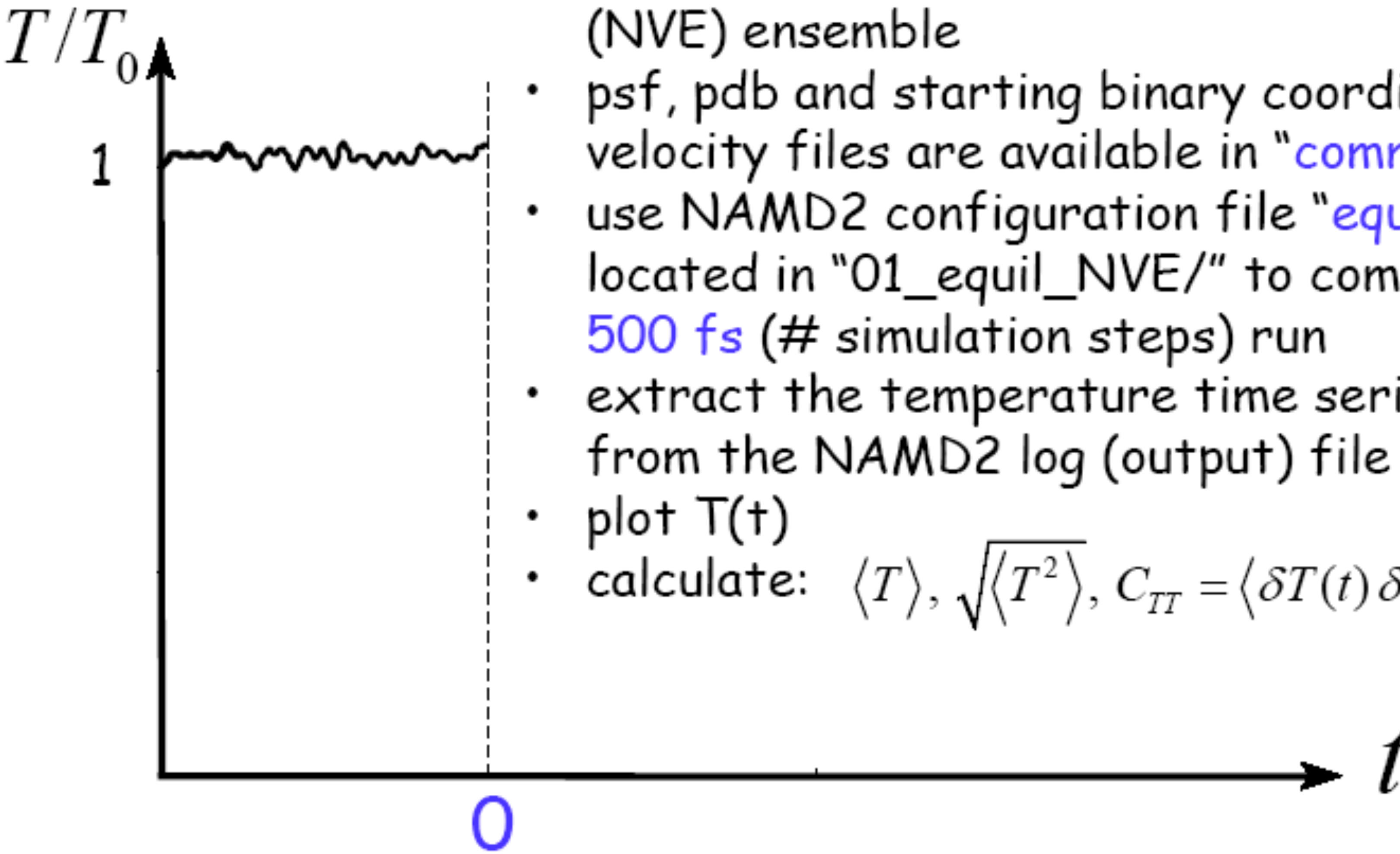
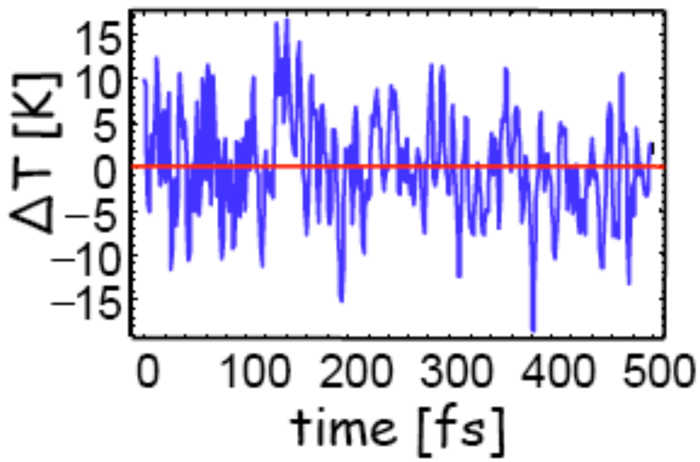
Generating T-Quench Echo: Step1

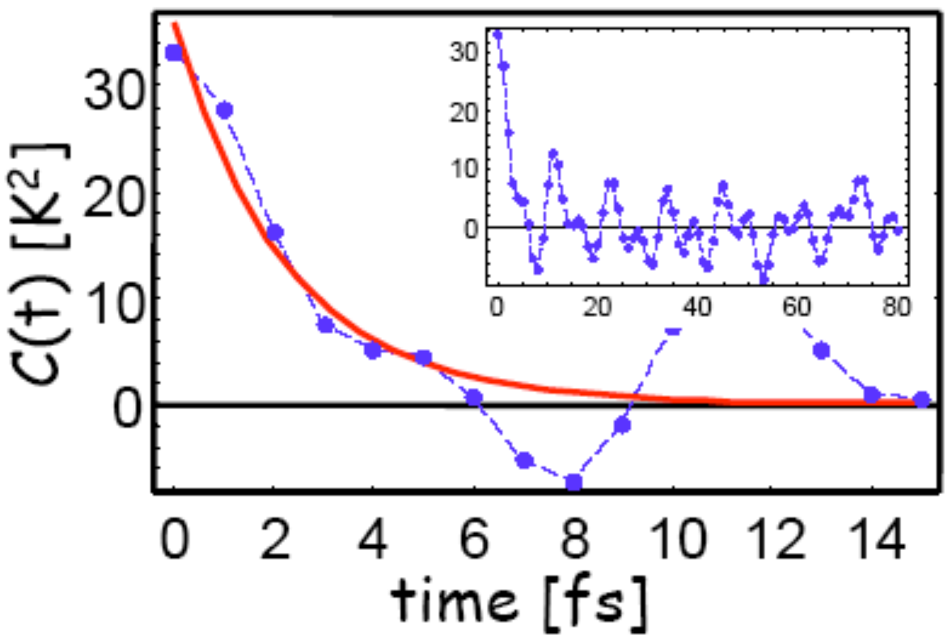
- your system is ubiquitin (1UBQ) in vacuum, pre-equilibrated at T_0=300K
- run all simulations in the *microcanonical* (NVE) ensemble
- psf, pdb and starting binary coordinate and velocity files are available in “common/”
- use NAMD2 configuration file “equil.conf” located in “01_equil_NVE/” to complete a 500 fs (# simulation steps) run
- extract the temperature time series T(t) from the NAMD2 log (output) file
- plot T(t)
- calculate: $\langle T \rangle, \sqrt{\langle T^2 \rangle}, C_{TT} = \langle \delta T(t)\, \delta T(0) \rangle$



Temperature Autocorrelation Function

$$\Delta T(t) = T(t) - \langle T(t) \rangle$$





$$C(t) = \langle \Delta T(t)\, \Delta T(0) \rangle$$

$$\rightarrow C(t_i) \approx \frac{1}{N-i} \sum_{n=1}^{N-i} \Delta T(t_{n+i})\, \Delta T(t_n)$$

$$C(t) = C(0) \exp\left(-t/\tau_0\right)$$

Temperature relaxation time:

$$\tau_0 \approx 2.2\ fs$$

Mean temperature:

$$\langle T \rangle = 299\,K$$

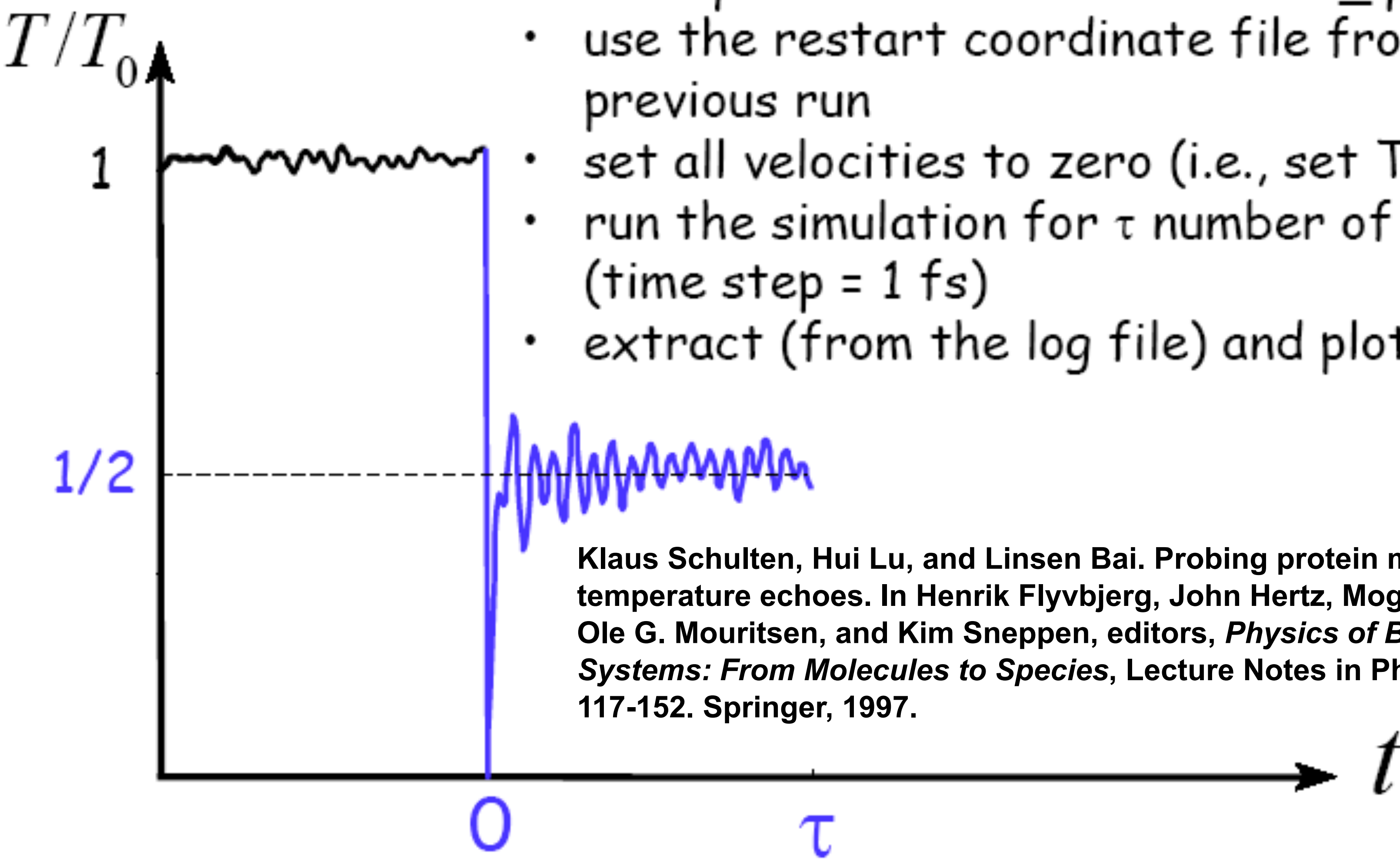
RMS temperature:

$$\sqrt{\langle \Delta T^2 \rangle} = \sqrt{C(0)} = 6\,K$$

Generating T-Quench Echo: Step2

Perform the 1st temperature quench

- start a new simulation using configuration file "quench.conf" located in "02_quencha/"
- use the restart coordinate file from the previous run
- set all velocities to zero (i.e., set T=0)
- run the simulation for τ number of steps (time step = 1 fs)
- extract (from the log file) and plot T(t)



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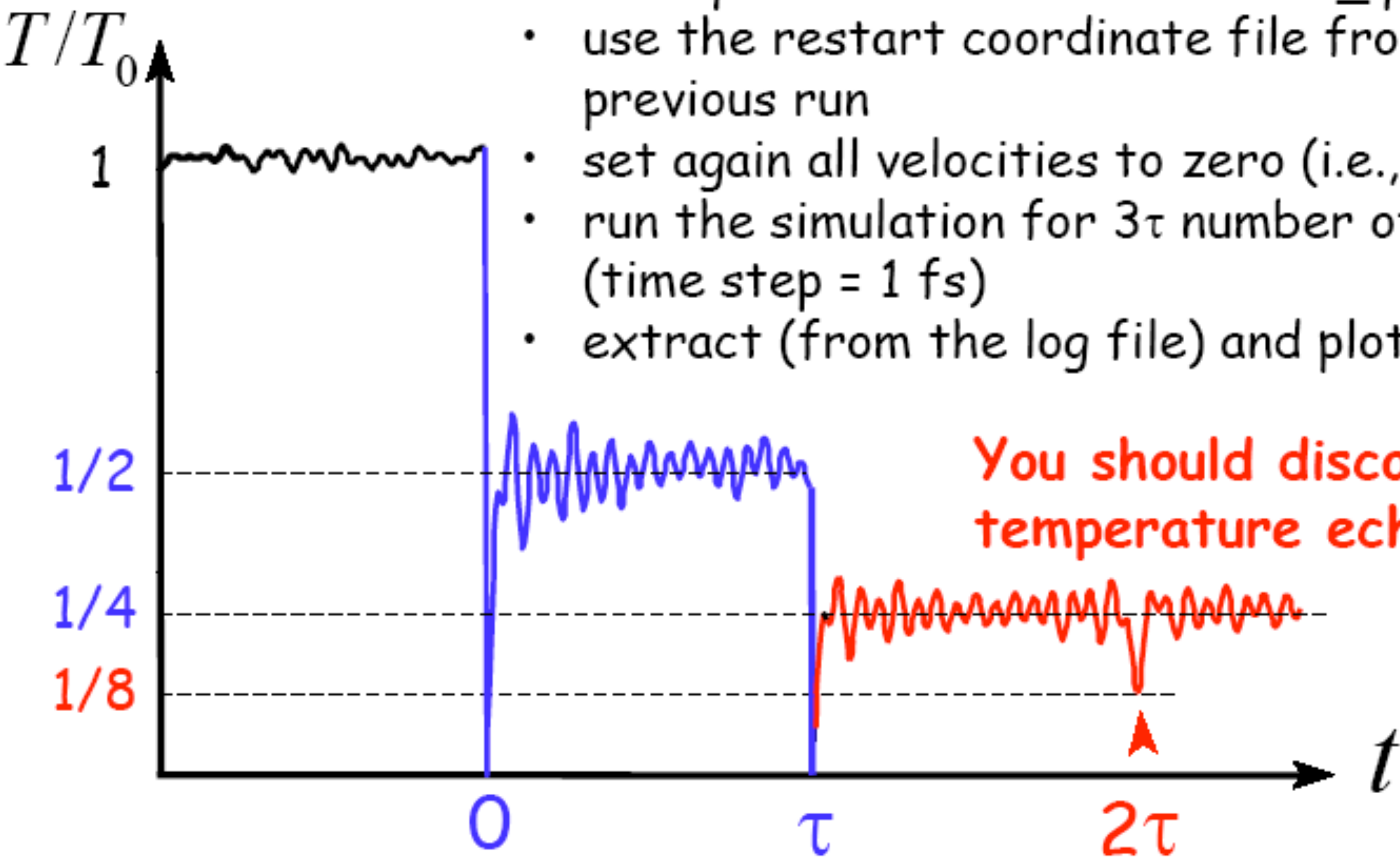
Generating T-Quench Echo: Step3

Perform the 2nd temperature quench

- start a new simulation using configuration file "quench.conf" located in "03_quenchb/"
- use the restart coordinate file from the previous run
- set again all velocities to zero (i.e., set T=0)
- run the simulation for 3τ number of steps (time step = 1 fs)
- extract (from the log file) and plot T(t)



Explanation of the T-Quench Echo

Assumption: protein ≈ collection of weakly interacting harmonic oscillators with dispersion $\omega = \omega_\alpha$, $\alpha = 1, \ldots, 3N-6$

Step1: $t < 0$

$$x(t) = A_0 \cos(\omega t + \theta_0)$$
$$v(t) = -\omega A_0 \sin(\omega t + \theta_0)$$

Step2: $0 < t < \tau$

$$\left.\begin{array}{l} x_1(t) = A_1 \cos(\omega t + \theta_1) \\ v_1(t) = -\omega A_1 \sin(\omega t + \theta_1) \end{array}\right\} \xrightarrow{v_1(0)=0} \left\{\begin{array}{l} A_1 = A_0 \cos\theta_0 \\ \theta_1 = 0 \end{array}\right.$$

Step3: $t > \tau$

$$\left.\begin{array}{l} x_2(t) = A_2 \cos(\omega t + \theta_2) \\ v_2(t) = -\omega A_2 \sin(\omega t + \theta_2) \end{array}\right\} \xrightarrow{v_2(\tau)=0} \left\{\begin{array}{l} A_2 = A_1 \cos\omega\tau \\ \theta_2 = -\omega\tau \end{array}\right.$$

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T-Quench Echo: Harmonic Approximation

$$T(t) \approx \frac{T_0}{4}\left[1 - \langle \cos(2\omega(t-\tau)) \rangle - \frac{1}{2}\langle \cos(2\omega(t-2\tau)) \rangle \right]$$

$$\approx \begin{cases} 0 & for\ t = \tau \\ T_0/8 & for\ t = 2\tau \\ T_0/4 & otherwise \end{cases}$$

$$\Rightarrow \text{echo depth } = T(2\tau) - T_0/4 = T_0/8$$

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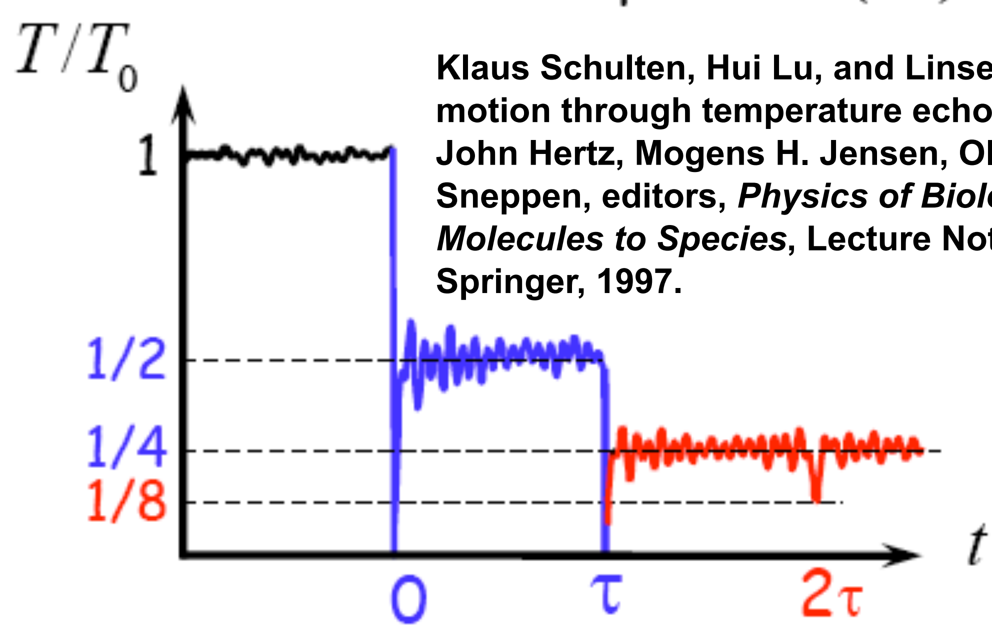

$T(t)$ and $C_{TT}(t)$

It can be shown:

$$\left\langle \cos(2\omega t)\right\rangle = \frac{\left\langle \delta T(t)\,\delta T(0)\right\rangle}{\left\langle \Delta T^2\right\rangle} = C_{TT}(t), \qquad \delta T(t) = T(t) - \left\langle T\right\rangle$$

Accordingly,

$$T(t) \approx \frac{T_0}{4}\left[1 - \langle\cos(2\omega(t-\tau))\rangle - \frac{1}{2}\langle\cos(2\omega(t-2\tau))\rangle\right]$$

$$\Downarrow$$

$$= \frac{T_0}{4}\left[1 - C_{TT}(t-\tau) - \frac{1}{2}C_{TT}(t-2\tau)\right]$$

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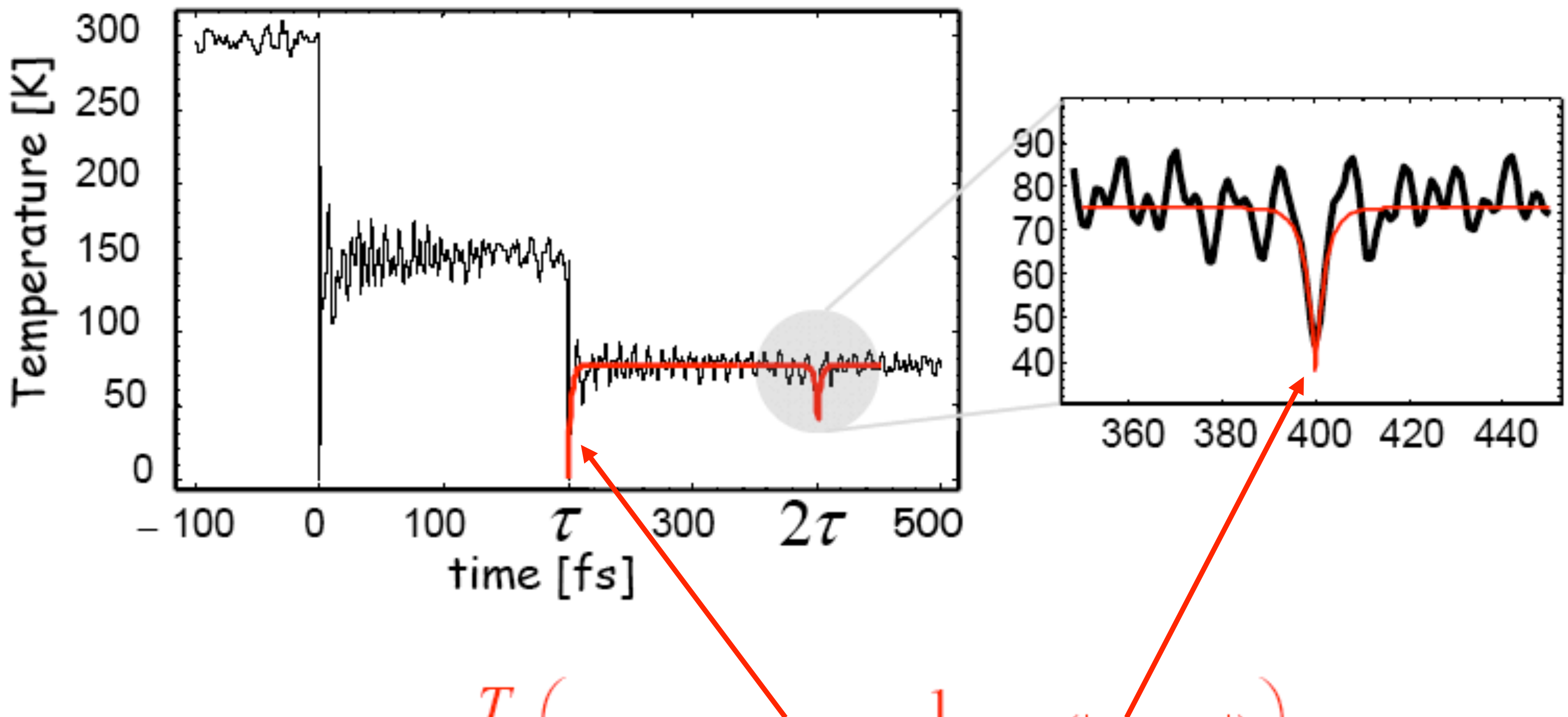
T-Quench Echo: Harmonic Approximation



$$T(t) \approx \frac{T_0}{2}\left(1 - C_{TT}(t-\tau) - \frac{1}{2} C_{TT}\left(|t-2\tau|\right)\right)$$

$$C_{TT}(t) = \exp\left(-t/\tau_0\right), \qquad \tau_0 \approx 2.2\ fs$$

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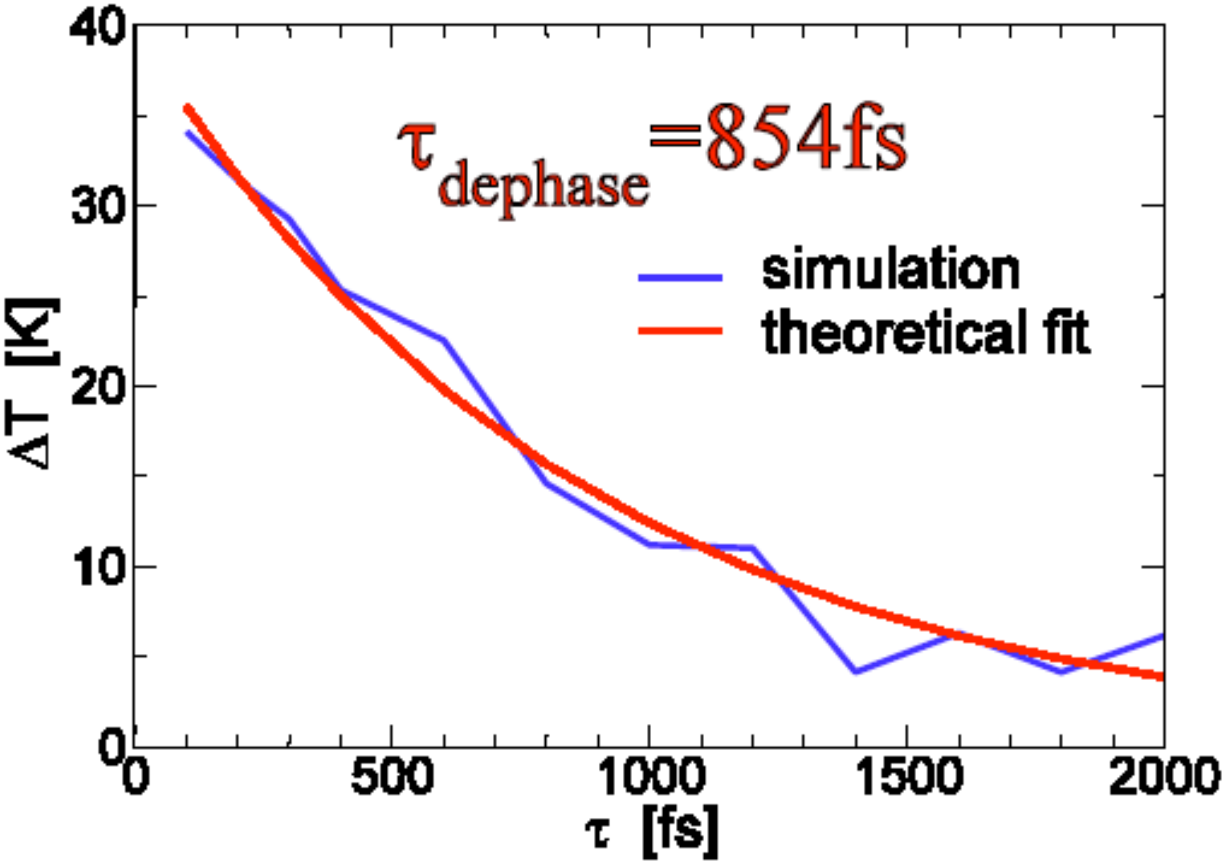
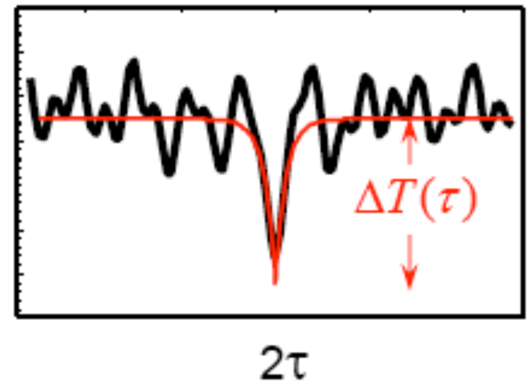
Dephasing Time of T-Quench Echoes





$$\Delta T(\tau) = \Delta T(0)\exp[-\tau / \tau_{dephase}]$$

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Constant Velocity Reassignment Echo ?

Can we get temperature echo(es) by reassigning the same set of atomic velocities (corresponding to T_0!) at $t = 0$ and $t = \tau$?

$$v_i\left(0^+\right) = v_i\left(\tau^+\right) = u_i\,,\;\; i = 1,\ldots,3N-6$$