2 Analysis

2.1 Equilibrium

2.1.1 RMSD for individual residues
2.1.2 Maxwell-Boltzmann Distribution
2.1.3 Energies
2.1.4 Temperature distribution
2.1.5 Specific Heat

2.2 Non-equilibrium properties of protein

2.2.1 Heat Diffusion

2.2.2 Temperature echoes
Temperature Echoes in Proteins

- Coherent motion in proteins: Echoes
- Generation of echoes in ubiquitin via velocity reassignments
  1) Temperature quench echoes
  2) Constant velocity reassignment echoes
  3) Velocity reassignment echoes

\[ T(t) = \frac{2}{(3N - 6)k_B} \sum_{n=1}^{3N-6} \frac{m_n v_n^2(t)}{2} \]
Temperature Echoes

- are sharp, resonance-like features in the time evolution of the protein’s temperature
- can be produced through 2 consecutive velocity reassignments

protein in equilibrium

synchronization signal
probing signal

temperature echoes
(a kind of “interference” effect!)
Velocity Reassignments

- Protein $\approx$ collection of weakly interacting harmonic oscillators having different frequencies
- At $t_1 = 0$ the 1st velocity reassignment: $v_i(0) = \lambda_1 u_i$ synchronizes the oscillators (i.e., make them oscillate in phase)
- At $t_2 = \tau$ (delay time) the 2nd velocity reassignment: $v_i(\tau) = \lambda_2 u_i$ probes the degree of coherence of the system at that moment
- Degree of coherence is characterized by:
  - The time(s) of the echo(es)
  - The depth of the echo(es)

$$\lambda_1 = \lambda_2 = 0 \implies \text{temperature quench}$$
$$\lambda_1 = \lambda_2 = 1 \implies \text{constant velocity reassignment}$$
$$\lambda_1 \neq \lambda_2 \neq 1 \implies \text{velocity reassignment}$$
Producing Temperature Echoes by
Velocity Reassignments in Proteins

Equilibration

No heat bath coupling: $T = T_0$

Equilibrium

No heat bath coupling

Relaxation

No heat bath coupling

Relaxation

No heat bath coupling

Temperature quench echoes:

$\nu_i(0) = \nu_i(\tau) = 0$

Const velocity reassignment echoes:

$\nu_i(0) = \nu_i(\tau) = u_i$

Velocity reassignment echoes:

$\nu_i(0) = u_i, \nu_i(\tau) = \lambda u_i$
Generating T-Quench Echo: Step 1

- your system is ubiquitin (1UBQ) in vacuum, pre-equilibrated at \( T_0 = 300 \text{K} \)
- run all simulations in the *microcanonical* (NVE) ensemble
- psf, pdb and starting binary coordinate and velocity files are available in “common/”
- use NAMD2 configuration file “equil.conf” located in “01_equil_NVE/” to complete a 500 fs (# simulation steps) run
- extract the temperature time series \( T(t) \) from the NAMD2 log (output) file
- plot \( T(t) \)
- calculate: \( \langle T \rangle, \sqrt{\langle T^2 \rangle}, C_{TT} = \langle \delta T(t) \delta T(0) \rangle \)
Temperature Autocorrelation Function

\[ \Delta T(t) = T(t) - \langle T(t) \rangle \]

\[
C(t) = \langle \Delta T(t) \Delta T(0) \rangle \\
\rightarrow C(t_i) \approx \frac{1}{N-i} \sum_{n=1}^{N-i} \Delta T(t_{n+i}) \Delta T(t_n)
\]

\[
C(t) = C(0) \exp\left(-t/\tau_0\right)
\]

Temperature relaxation time:
\[ \tau_0 \approx 2.2 \text{ fs} \]

Mean temperature:
\[ \langle T \rangle = 299 \text{ K} \]

RMS temperature:
\[ \sqrt{\langle \Delta T^2 \rangle} = \sqrt{C(0)} = 6 \text{ K} \]
Perform the 1\textsuperscript{st} temperature quench
- start a new simulation using configuration file “quench.conf” located in “02_quencha/”
- use the restart coordinate file from the previous run
- set all velocities to zero (i.e., set T=0)
- run the simulation for \( \tau \) number of steps (time step = 1 fs)
- extract (from the log file) and plot \( T(t) \)
Perform the 2\textsuperscript{nd} temperature quench
- start a new simulation using configuration file “quench.conf” located in “03_quenchr/”
- use the restart coordinate file from the previous run
- set again all velocities to zero (i.e., set $T=0$)
- run the simulation for $3\tau$ number of steps (time step = 1 fs)
- extract (from the log file) and plot $T(t)$

You should discover a temperature echo at $2\tau$
**Explanation of the T-Quench Echo**

**Assumption**: protein \( \approx \) collection of weakly interacting harmonic oscillators with dispersion \( \omega = \omega_{\alpha} \), \( \alpha = 1, \ldots, 3N - 6 \)

**Step 1**: \( t < 0 \)
\[
x(t) = A_0 \cos(\omega t + \theta_0)
\]
\[
v(t) = -\omega A_0 \sin(\omega t + \theta_0)
\]

**Step 2**: \( 0 < t < \tau \)
\[
x_1(t) = A_1 \cos(\omega t + \theta_1)
\]
\[
v_1(t) = -\omega A_1 \sin(\omega t + \theta_1)
\]
\[
\begin{align*}
v_1(0) &= 0 \\
A_1 &= A_0 \cos \theta_0 \\
\theta_1 &= 0
\end{align*}
\]

**Step 3**: \( t > \tau \)
\[
x_2(t) = A_2 \cos(\omega t + \theta_2)
\]
\[
v_2(t) = -\omega A_2 \sin(\omega t + \theta_2)
\]
\[
\begin{align*}
v_2(\tau) &= 0 \\
A_2 &= A_1 \cos \omega \tau \\
\theta_2 &= -\omega \tau
\end{align*}
\]
The average must be taken over the distribution of initial phases $\theta_0$, amplitudes $A_0$ and angular velocities $\omega$.

The equipartition theorem yields:

$$
\langle A_0^2 \cos^2 \theta_0 \rangle = \frac{1}{2} \langle A_0^2 \rangle = \frac{k_B T_0}{2 m \omega^2}
$$

Thus:

$$
T(t) = T_0 \left[ 1 + \langle \cos(2\omega \tau) \rangle - \langle \cos(2\omega(t-\tau)) \rangle - \frac{1}{2} \langle \cos(2\omega t) \rangle - \frac{1}{2} \langle \cos(2\omega(t-2\tau)) \rangle \right]
$$

Since: $\langle \cos(\omega?) \rangle_\omega \approx 0$ unless $? = 0 \Rightarrow$
**T-Quench Echo: Harmonic Approximation**

\[
T(t) \approx \frac{T_0}{4} \left[ 1 - \langle \cos(2\omega(t - \tau)) \rangle - \frac{1}{2} \langle \cos(2\omega(t - 2\tau)) \rangle \right]
\]

\[
\approx \begin{cases} 
0 & \text{for } t = \tau \\
\frac{T_0}{8} & \text{for } t = 2\tau \\
\frac{T_0}{4} & \text{otherwise}
\end{cases}
\]

\[\Rightarrow \text{echo depth } = T(2\tau) - \frac{T_0}{4} = \frac{T_0}{8}\]
\(T(t)\) and \(C_{TT}(t)\)

It can be shown that:

\[
\langle \cos(2\omega t) \rangle = \frac{\langle \delta T(t) \delta T(0) \rangle}{\langle \Delta T^2 \rangle} = C_{TT}(t), \quad \delta T(t) = T(t) - \langle T \rangle
\]

\[
\downarrow
\]

\[
T(t) = \frac{T_0}{4} \left[ 1 + C_{TT}(\tau) - C_{TT}(t-\tau) - \frac{1}{2} C_{TT}(t) - \frac{1}{2} C_{TT}(|t-2\tau|) \right]
\]

\[
\approx \frac{T_0}{4} \left[ 1 - \frac{1}{2} C_{TT}(|t-2\tau|) \right] \quad \text{for} \quad t > \tau
\]
$T(t) \approx \frac{T_0}{2} \left(1 - C_{TT}(t - \tau) - \frac{1}{2} C_{TT}(|t - 2\tau|)\right)$

$C_{TT}(t) = \exp\left(-t / \tau_0\right), \quad \tau_0 \approx 2.2 \text{ fs}$
Dephasing Time of T-Quench Echoes

\[ \Delta T(\tau) = \Delta T(0) \exp\left[-\frac{\tau}{\tau_{\text{dephase}}}\right] \]

![Graph showing the dephasing process with \( \tau_{\text{dephase}} = 854 \text{fs} \). The graph compares simulation (blue) and theoretical fit (red) against the theoretical fit. The dephasing time is quantified by \( \Delta T(\tau) \).]
Can we get temperature echo(es) by reassigning the same set of atomic velocities (corresponding to $T_0$!) at $t=0$ and $t=\tau$?

$$v_i(0^+) = v_i(\tau^+) = u_i, \ i = 1,...,3N-6$$

Answer: YES!

$$T(t) \approx T_0 \left[ 1 - \frac{1}{2} C_{TT} (|t - 3\tau/2|) \right]$$
Is it possible to produce temperature echo with a single velocity reassignment?

YES!

Reset all velocities at time $\tau$ to the values at a previous instant of time, i.e., $t=0$