

NAMD Tutorial (Part 2)

▶ 2 Analysis

▶ 2.1 Equilibrium

- ▶ 2.1.1 RMSD for individual residues
- ▶ 2.1.2 Maxwell-Boltzmann Distribution
- ▶ 2.1.3 Energies
- ▶ 2.1.4 Temperature distribution
- ▶ 2.1.5 Specific Heat

▶ 2.2 Non-equilibrium properties of protein

- ▶ 2.2.1 Heat Diffusion
- ▶ 2.2.2 Temperature echoes

Temperature Echoes in Proteins

- ▶ Coherent motion in proteins: Echoes
- ▶ Generation of echoes in *ubiquitin* via velocity reassignments
 - 1) Temperature quench echoes
 - 2) Constant velocity reassignment echoes
 - 3) Velocity reassignment echoes

temperature $\Leftrightarrow$ velocities

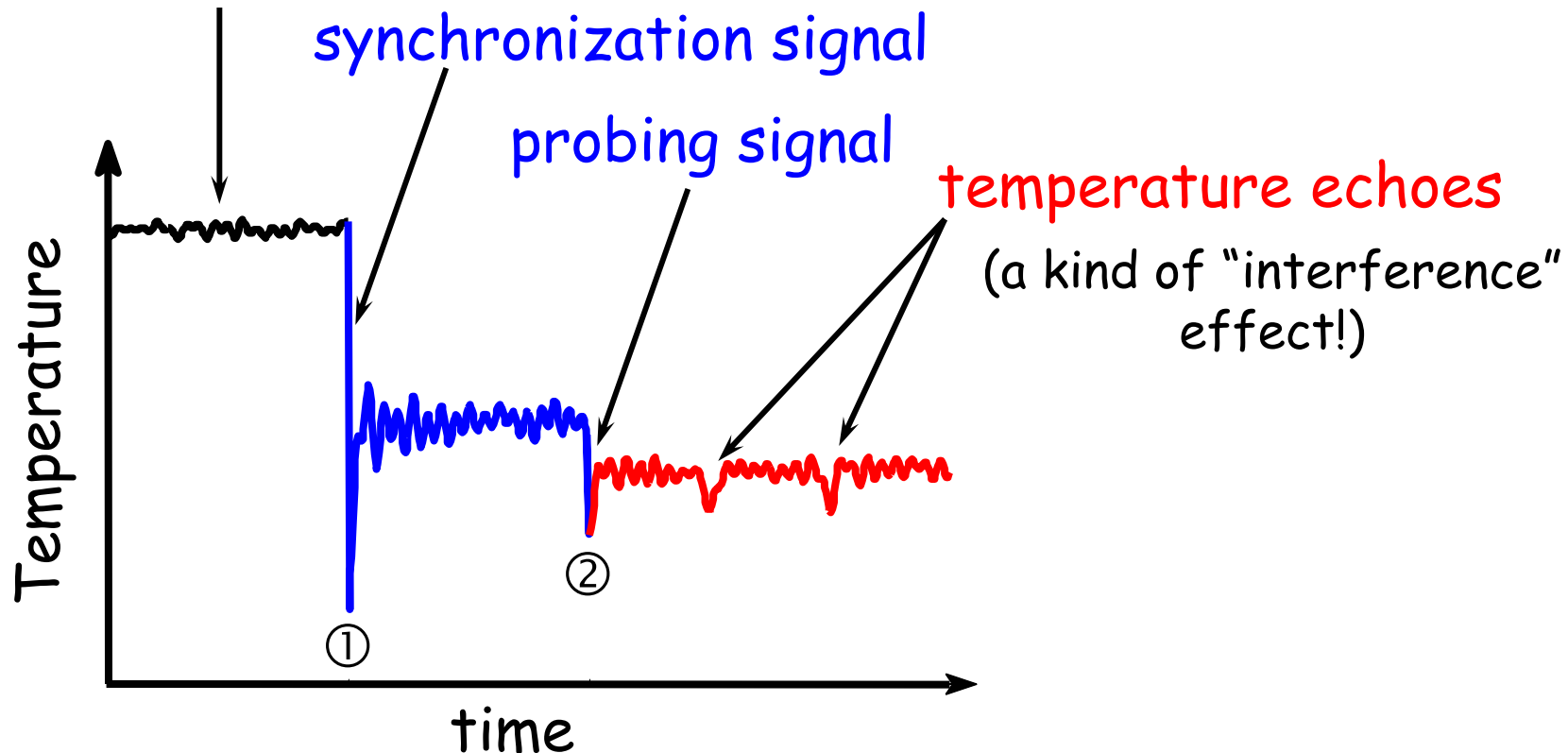
kinetic temperature:

$$T(t) = \frac{2}{(3N - 6)k_B} \sum_{n=1}^{3N-6} \frac{m_n v_n^2(t)}{2}$$

Temperature Echoes

- are sharp, resonance-like features in the time evolution of the protein's temperature
- can be produced through 2 consecutive velocity reassignments

protein in equilibrium



Velocity Reassignments

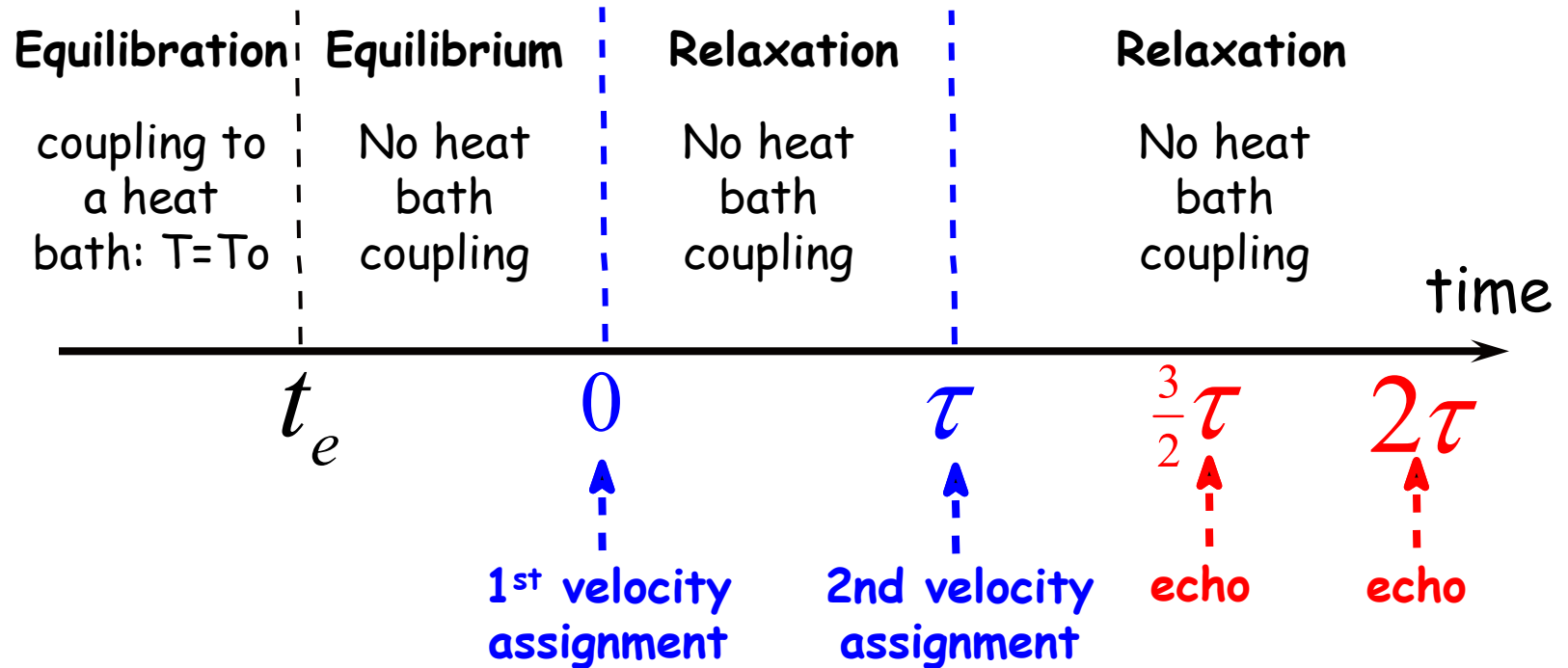
- ▶ protein $\approx$ collection of weakly interacting harmonic oscillators having different frequencies
- ▶ at $t_1=0$ the 1st velocity reassignment: $v_i(0)=\lambda_1 u_i$ synchronizes the oscillators (i.e., make them oscillate in phase)
- ▶ at $t_2=\tau$ (delay time) the 2nd velocity reassignment: $v_i(\tau)=\lambda_2 u_i$ probes the degree of coherence of the system at that moment
- ▶ degree of coherence is characterized by:
 - the time(s) of the echo(es)
 - the depth of the echo(es)

$\lambda_1 = \lambda_2 = 0 \Rightarrow$ temperature quench

$\lambda_1 = \lambda_2 = 1 \Rightarrow$ constant velocity reassignment

$\lambda_1 \neq \lambda_2 \neq 1 \Rightarrow$ velocity reassignment

Producing Temperature Echoes by Velocity Reassignments in Proteins

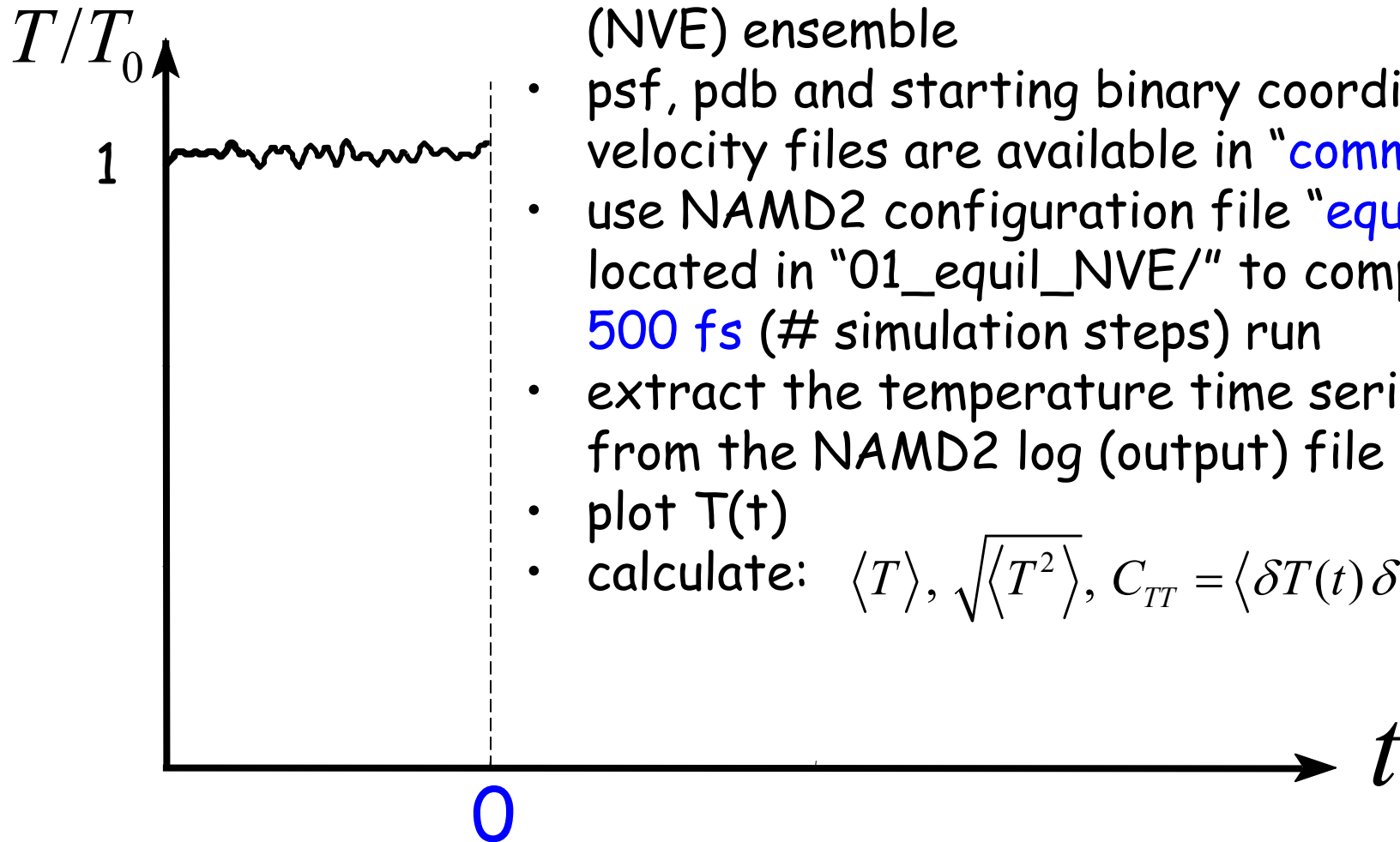


Temperature quench echoes: $v_i(0) = v_i(\tau) = 0$

Const velocity reassignment echoes: $v_i(0) = v_i(\tau) = u_i$

Velocity reassignment echoes: $v_i(0) = u_i, v_i(\tau) = \lambda u_i$

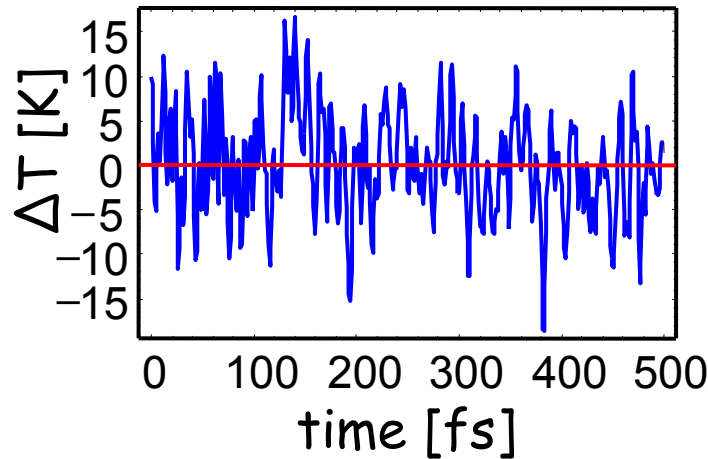
Generating T-Quench Echo: Step1



- your system is ubiquitin (1UBQ) in vacuum, pre-equilibrated at $T_0=300\text{K}$
- run all simulations in the *microcanonical* (NVE) ensemble
- psf, pdb and starting binary coordinate and velocity files are available in "common/"
- use NAMD2 configuration file "equil.conf" located in "01_equil_NVE/" to complete a 500 fs (# simulation steps) run
- extract the temperature time series $T(t)$ from the NAMD2 log (output) file
- plot $T(t)$
- calculate: $\langle T \rangle$, $\sqrt{\langle T^2 \rangle}$, $C_{TT} = \langle \delta T(t) \delta T(0) \rangle$

Temperature Autocorrelation Function

$$\Delta T(t) = T(t) - \langle T(t) \rangle$$



$$C(t) = \langle \Delta T(t) \Delta T(0) \rangle$$

$$\rightarrow C(t_i) \approx \frac{1}{N-i} \sum_{n=1}^{N-i} \Delta T(t_{n+i}) \Delta T(t_n)$$

$$C(t) = C(0) \exp(-t/\tau_0)$$

Temperature relaxation time:

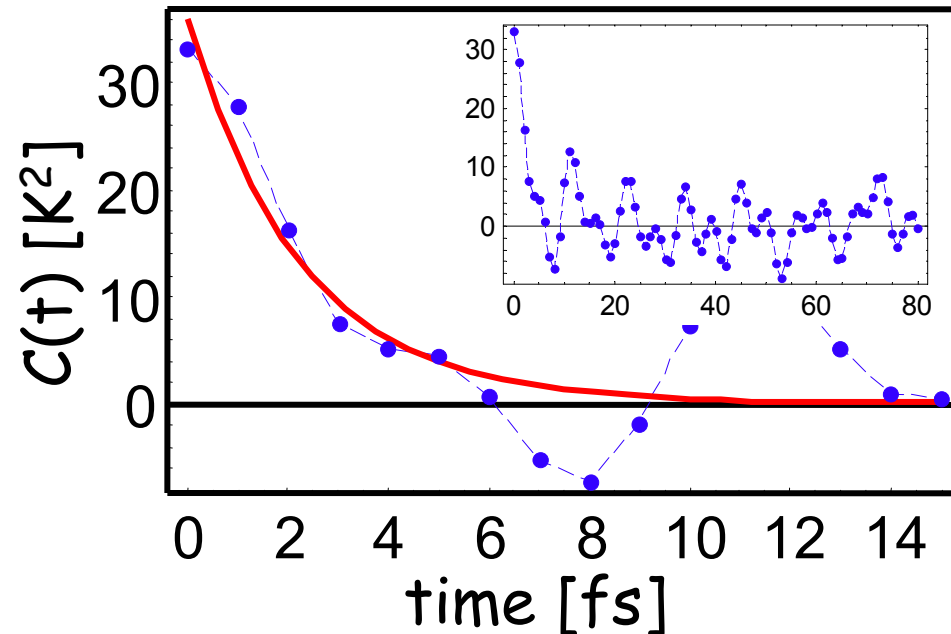
$$\tau_0 \approx 2.2 \text{ fs}$$

Mean temperature:

$$\langle T \rangle = 299 \text{ K}$$

RMS temperature:

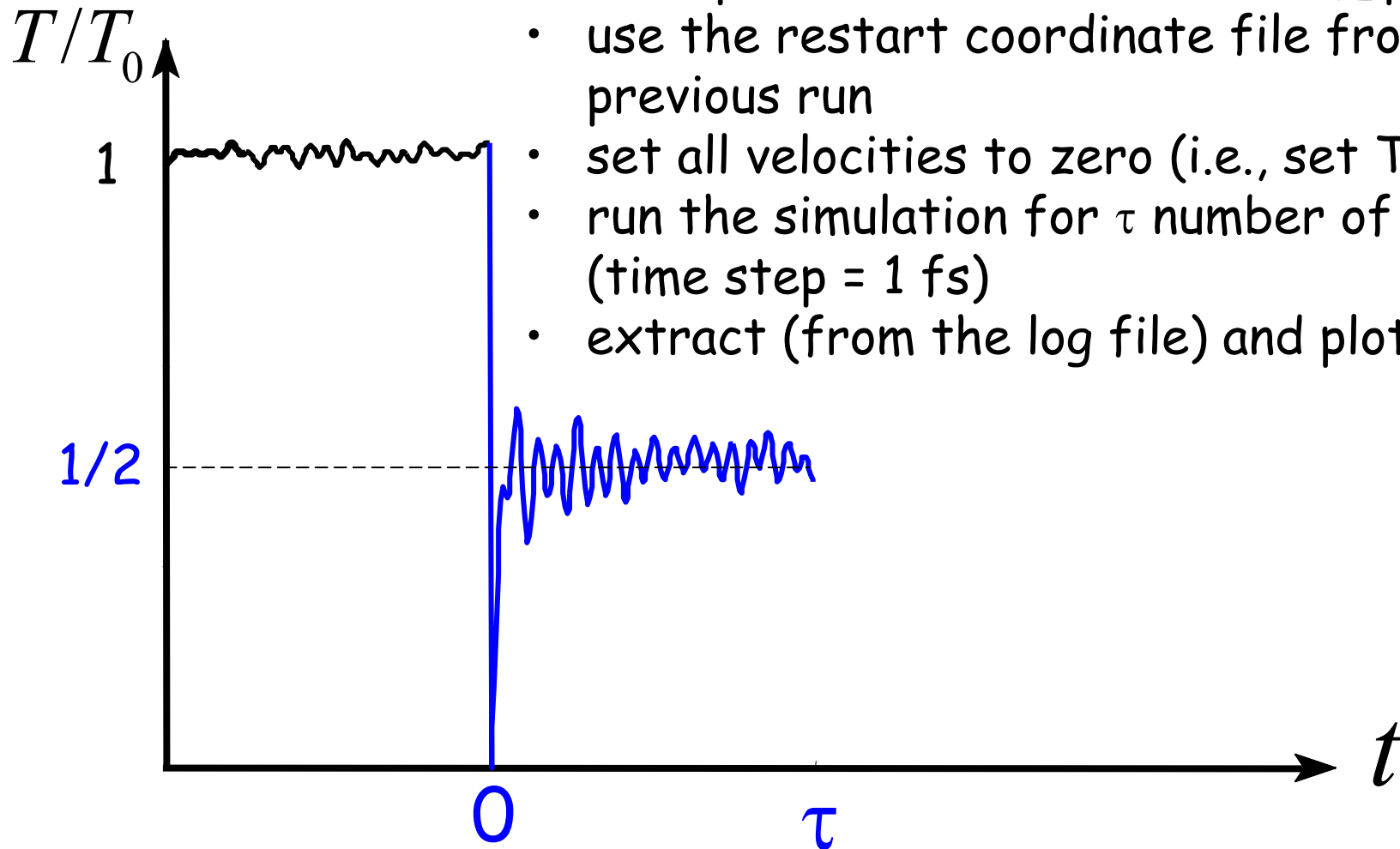
$$\sqrt{\langle \Delta T^2 \rangle} = \sqrt{C(0)} = 6 \text{ K}$$



Generating T-Quench Echo: Step2

Perform the 1st temperature quench

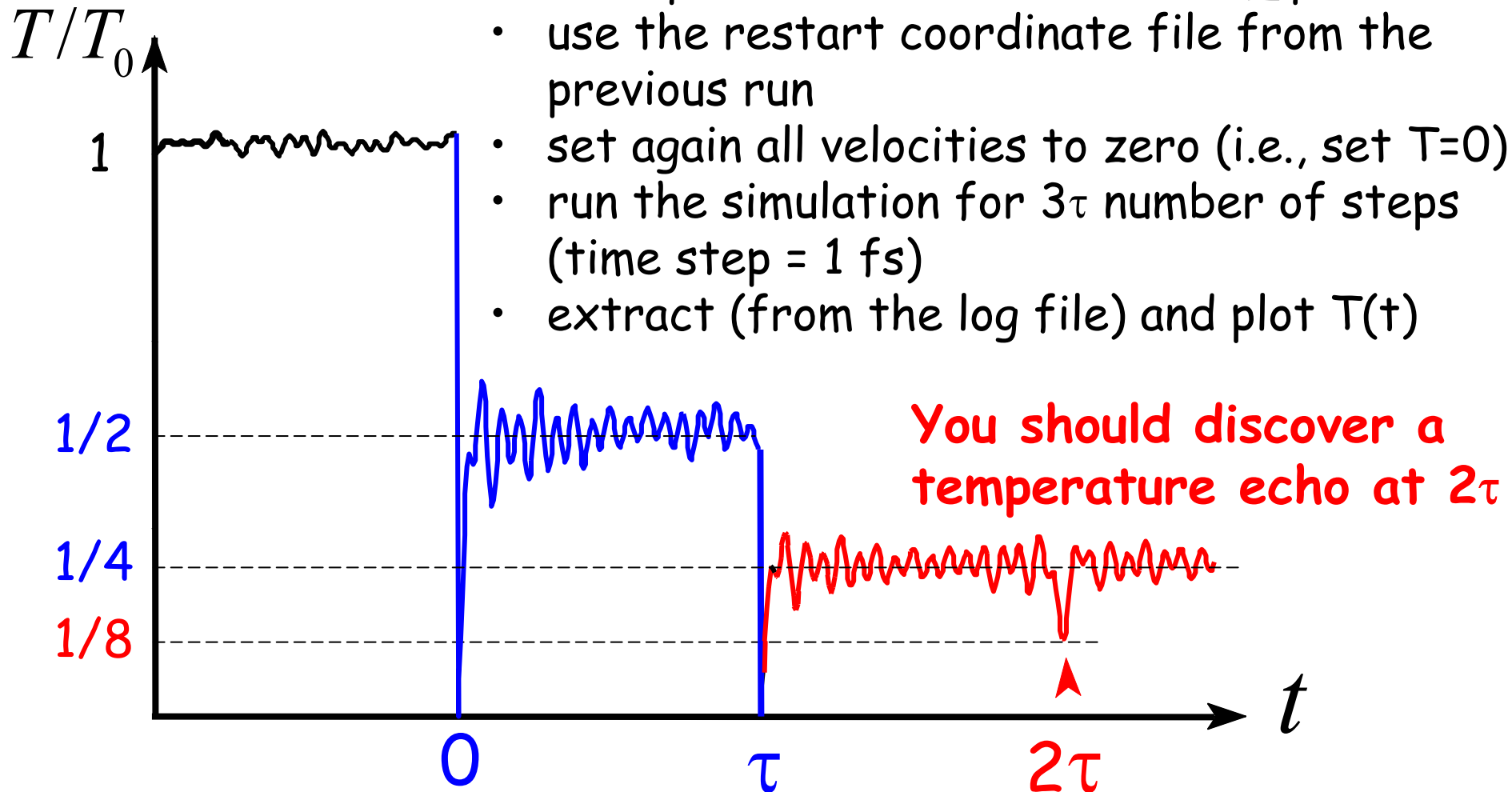
- start a new simulation using configuration file "quench.conf" located in "02_quencha/"
- use the restart coordinate file from the previous run
- set all velocities to zero (i.e., set $T=0$)
- run the simulation for τ number of steps (time step = 1 fs)
- extract (from the log file) and plot $T(t)$



Generating T-Quench Echo: Step3

Perform the 2nd temperature quench

- start a new simulation using configuration file "quench.conf" located in "03_quenchb/"
- use the restart coordinate file from the previous run
- set again all velocities to zero (i.e., set $T=0$)
- run the simulation for 3τ number of steps (time step = 1 fs)
- extract (from the log file) and plot $T(t)$



Explanation of the T-Quench Echo

Assumption: protein $\approx$ collection of weakly interacting harmonic oscillators with dispersion $\omega = \omega_\alpha$, $\alpha = 1, \dots, 3N - 6$

Step1: $t < 0$

$$\begin{aligned} x(t) &= A_0 \cos(\omega t + \theta_0) \\ v(t) &= -\omega A_0 \sin(\omega t + \theta_0) \end{aligned}$$

Step2: $0 < t < \tau$

$$\left. \begin{aligned} x_1(t) &= A_1 \cos(\omega t + \theta_1) \\ v_1(t) &= -\omega A_1 \sin(\omega t + \theta_1) \end{aligned} \right\} \xrightarrow{v_1(0)=0} \begin{cases} A_1 = A_0 \cos \theta_0 \\ \theta_1 = 0 \end{cases}$$

Step3: $t > \tau$

$$\left. \begin{aligned} x_2(t) &= A_2 \cos(\omega t + \theta_2) \\ v_2(t) &= -\omega A_2 \sin(\omega t + \theta_2) \end{aligned} \right\} \xrightarrow{v_2(\tau)=0} \begin{cases} A_2 = A_1 \cos \omega \tau \\ \theta_2 = -\omega \tau \end{cases}$$

T-Quench Echo: Harmonic Approximation

for $t > \tau$:
$$T(t) \propto \langle v_2^2 \rangle = \langle \omega^2 A_0^2 \cos^2 \theta_0 \cos^2(\omega \tau) \sin^2(\omega(t - \tau)) \rangle$$

The average must be taken over the distribution of initial phases θ_0 , amplitudes A_0 and angular velocities ω

equipartition theorem $\Rightarrow \langle A_0^2 \cos^2 \theta_0 \rangle = \frac{1}{2} \langle A_0^2 \rangle = \frac{k_B T_0}{2 m \omega^2}$

$$\begin{aligned} T(t) &= T_0 \langle \cos^2(\omega \tau) \sin^2(\omega(t - \tau)) \rangle = \dots \\ &= \frac{T_0}{4} \left[1 + \langle \cos(2\omega \tau) \rangle - \langle \cos(2\omega(t - \tau)) \rangle \right. \\ &\quad \left. - \frac{1}{2} \langle \cos(2\omega t) \rangle - \frac{1}{2} \langle \cos(2\omega(t - 2\tau)) \rangle \right] \end{aligned}$$

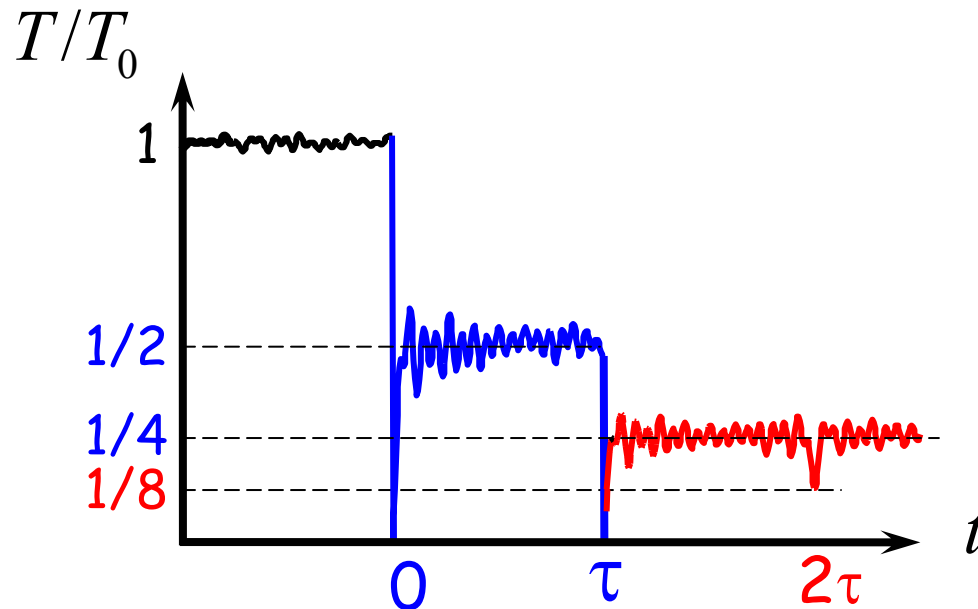
Since : $\langle \cos(\omega?) \rangle_\omega \approx 0$ unless $? = 0 \Rightarrow$

T-Quench Echo: Harmonic Approximation

$$T(t) \approx \frac{T_0}{4} \left[1 - \langle \cos(2\omega(t - \tau)) \rangle - \frac{1}{2} \langle \cos(2\omega(t - 2\tau)) \rangle \right]$$

$$\approx \begin{cases} 0 & \text{for } t = \tau \\ T_0/8 & \text{for } t = 2\tau \\ T_0/4 & \text{otherwise} \end{cases}$$

$$\Rightarrow \text{echo depth} = T(2\tau) - T_0/4 = T_0/8$$



$T(t)$ and $C_{TT}(t)$

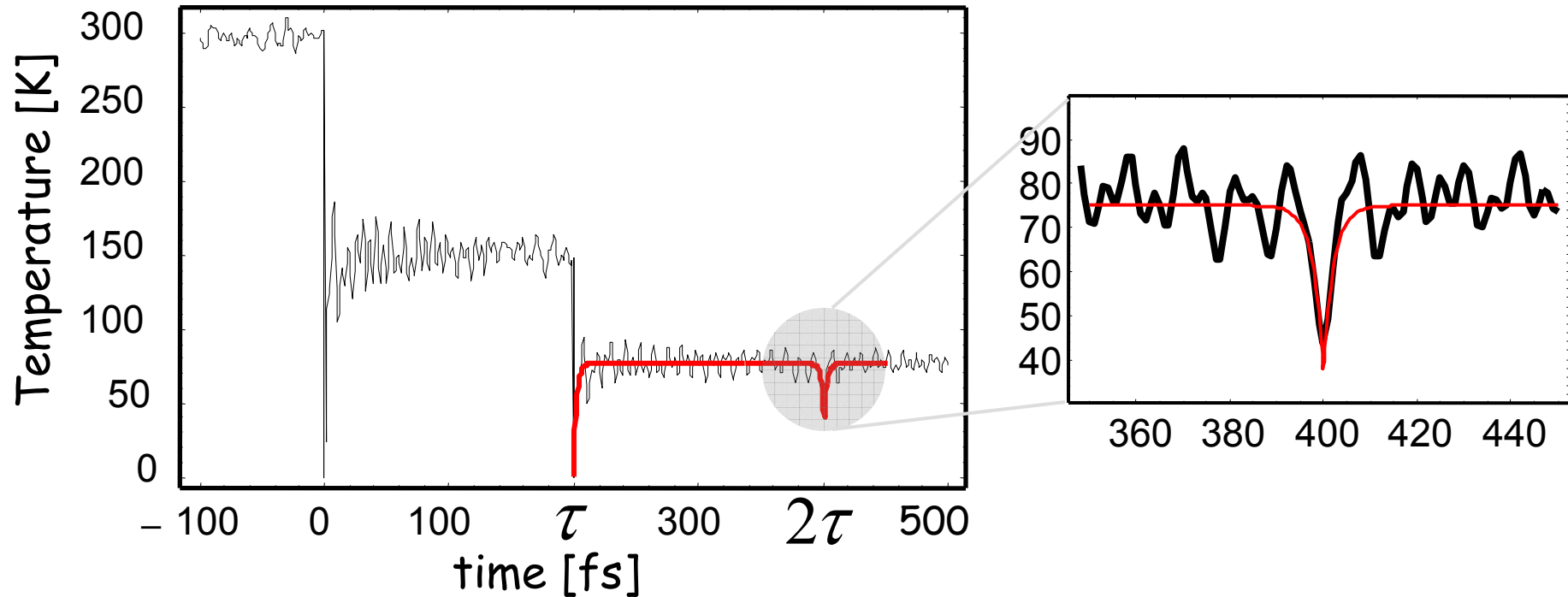
It can be shown that:

$$\langle \cos(2\omega t) \rangle = \frac{\langle \delta T(t) \delta T(0) \rangle}{\langle \Delta T^2 \rangle} = C_{TT}(t), \quad \delta T(t) = T(t) - \langle T \rangle$$

$\Downarrow$

$$T(t) = \frac{T_0}{4} \left[1 + C_{TT}(\tau) - C_{TT}(t - \tau) - \frac{1}{2} C_{TT}(t) - \frac{1}{2} C_{TT}(|t - 2\tau|) \right]$$
$$\approx \frac{T_0}{4} \left[1 - \frac{1}{2} C_{TT}(|t - 2\tau|) \right] \quad \text{for } t > \tau$$

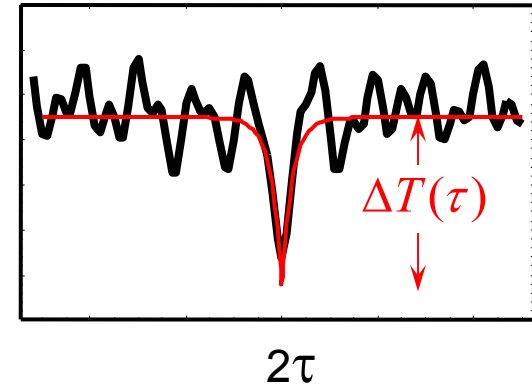
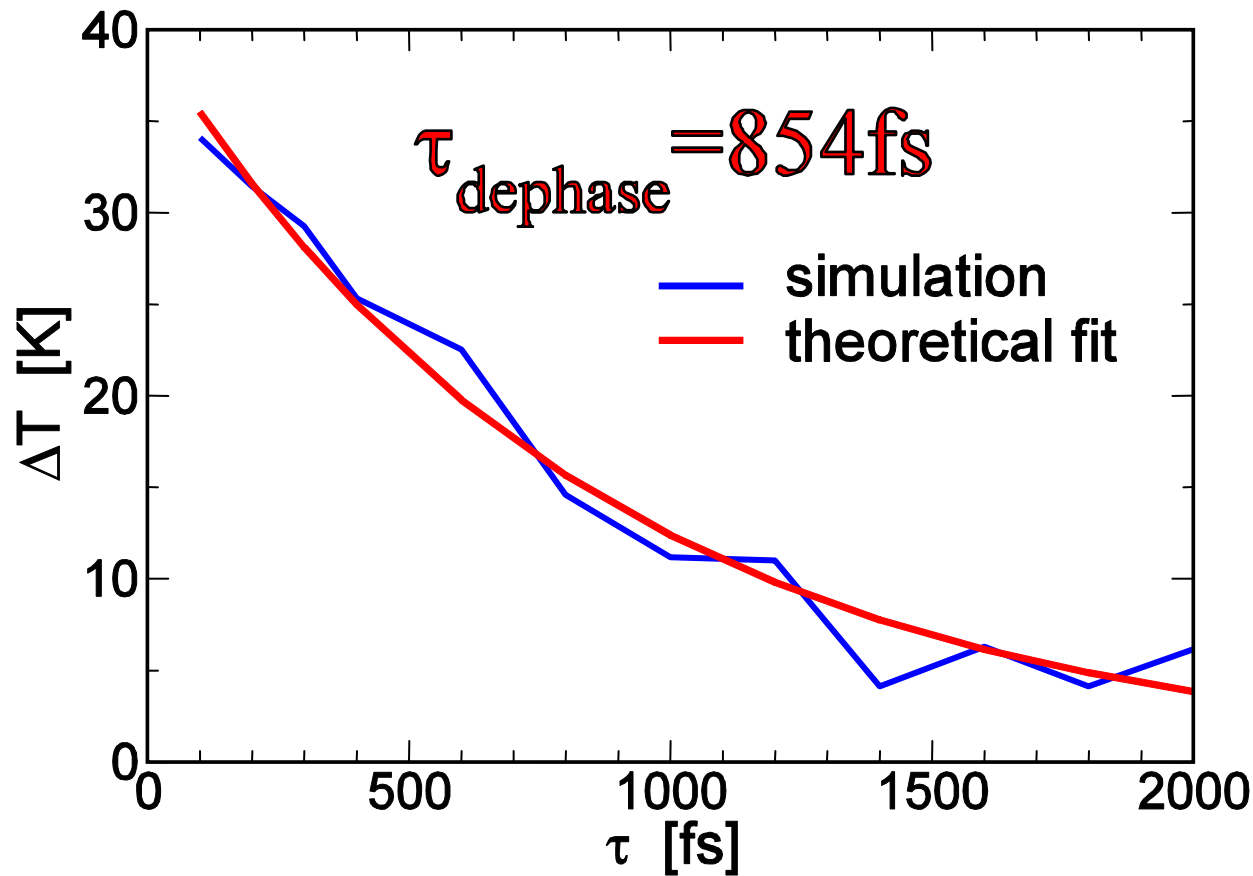
T-Quench Echo: Harmonic Approximation



$$T(t) \approx \frac{T_0}{2} \left(1 - C_{TT}(t - \tau) - \frac{1}{2} C_{TT}(|t - 2\tau|) \right)$$

$$C_{TT}(t) = \exp(-t / \tau_0), \quad \tau_0 \approx 2.2 \text{ fs}$$

Dephasing Time of T-Quench Echoes

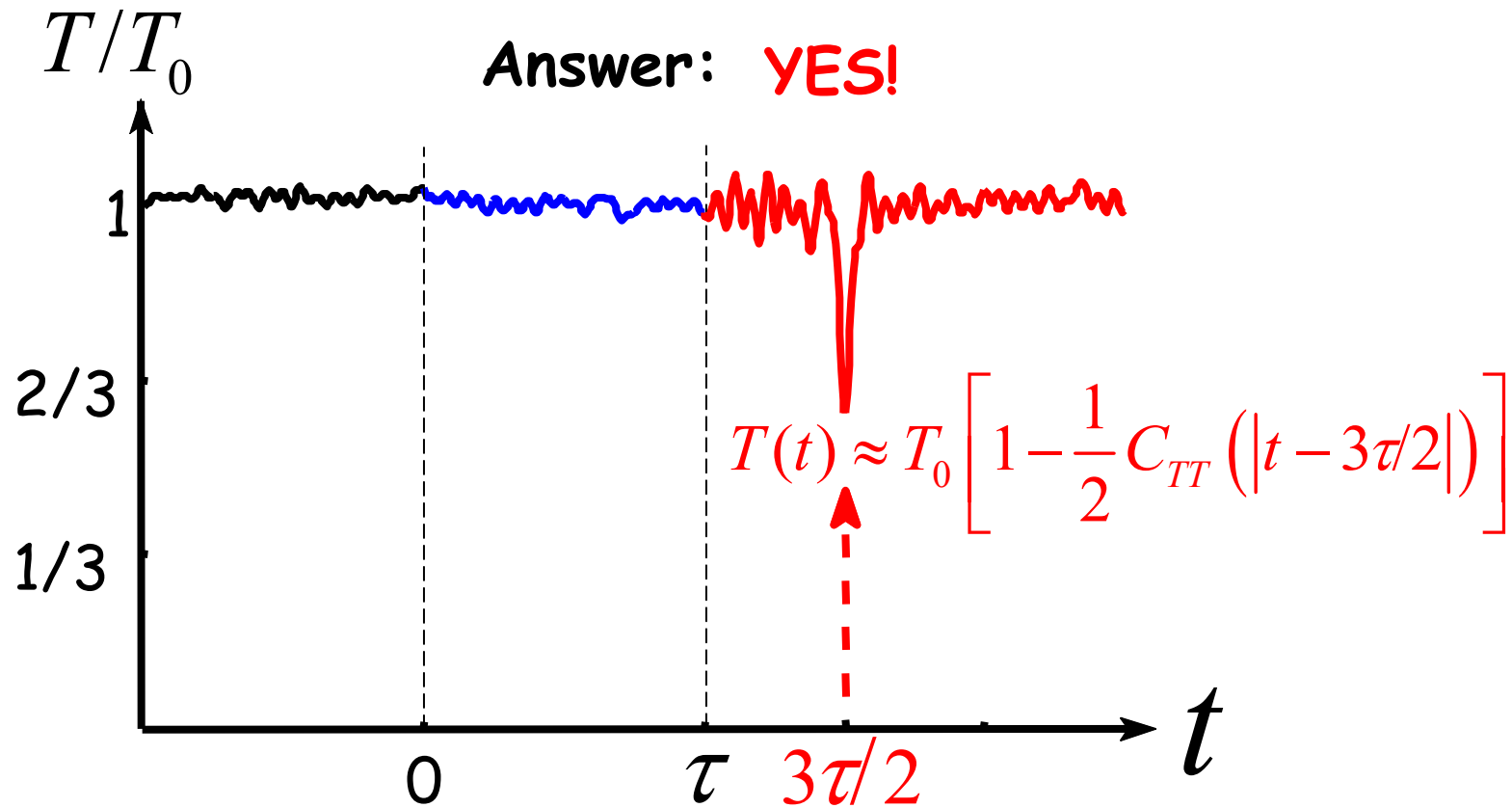


$$\Delta T(\tau) = \Delta T(0) \exp[-\tau / \tau_{\text{dephase}}]$$

Constant Velocity Reassignment Echo ?

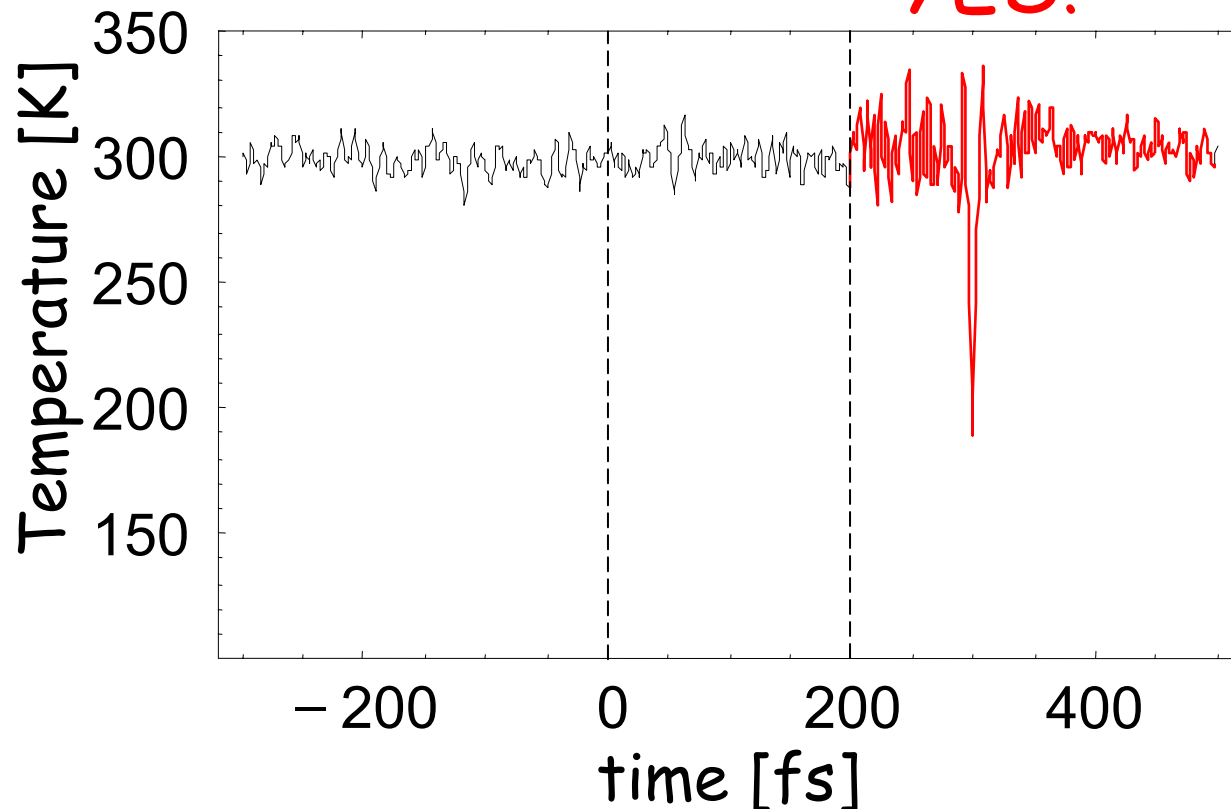
Can we get temperature echo(es) by reassigning the same set of atomic velocities (corresponding to T_0 !) at $t=0$ and $t=\tau$?

$$v_i(0^+) = v_i(\tau^+) = u_i, \quad i = 1, \dots, 3N-6$$



Is it possible to produce temperature echo with a single velocity reassignment ?

YES!



Reset all velocities at time τ to the values at a previous instant of time, i.e., $t=0$