Simulating Conformational Fluctuations



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LH2 in membrane: **85,000 atoms;** simulated for 2ns with NAMD2; NpT ensemble; periodic boundary condition; full electrostatics (PME)







The Effect of Static Disorder

Stochastic Hamiltonian



$p(\epsilon_{\alpha}) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-(\epsilon_{\alpha} - \epsilon)^2/2\sigma^2\right]$

also term project





forbidden states become allowed

Random Matrix Theory (RMT) and Spectral Fluctuations



Spectral density for 2N=8 (arbitrary units)

Analytical Computation of Spectral Density in Random Matrix Theory



integral over a small supermatrix.

The Effect of Dynamic Disorder



Excitations of Individual Chlorophy





Absorption spectrum of monomeric BChl a in ether. The spectrum shows four peaks, corresponding to the Qy transition at 773 nm, the Qx transition at 577 nm, and the By and Bx transitions at 391 and 358 nm, respectively. Structure of BChl a . R denotes the pyhtyl tail of BChl. The arrows indicate the transition dipole moments for the Q y (from pyrrol ring II t IV) and the Q x (from pyrrol ring I t III)t ransitions.

Two State QM System Coupled to Vibrational Mode

First case: Coupling to a single oscillator

Hamiltonian
$$\hat{H}_{qo}^{(s)} = \begin{pmatrix} \hat{H}_r^{(s)} & v \\ v & \hat{H}_p^{(s)} + E \end{pmatrix}$$

Protein matrix is a bath of oscillators linearly
coupled to the electron transfer according to
 $\hat{H}_r^{(s)} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 q^2$
 $\hat{H}_p^{(s)} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 \left(q - \frac{c}{m\omega^2}\right)^2$
only a single oscillator considered at present!

Eigenstates and Propagator of (Shifted) Oscillator

osc shi	<mark>illator</mark> fted oscilla	tor	$\hat{H}_r^{(s)}$ $\hat{H}_p^{(s)}$	$= \frac{1}{2}$ $= \frac{1}{2}$	$\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2$ $\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2$	$\frac{c^2q^2}{c^2}\left(q - \frac{c}{m\omega^2}\right)$	2
oscillator	$\langle q n angle^{({ m r})}$	$= \phi_n^{(\mathbf{r})}(\mathbf{q})$	$q) = \left(\frac{\lambda}{\pi}\right)$	$\int_{-\infty}^{\frac{1}{4}} (2^n r)$	$(n!)^{-\frac{1}{2}}H_n(\sqrt{\lambda})$	$q) e^{-\frac{1}{2}\lambda q^2}$	
eigenstates	$\epsilon_n^{(\mathrm{r})}$	$= \hbar\omega(n$	$(+\frac{1}{2})$	1	$\lambda = m\omega/\hbar$		
shifted oscilla	$\langle q n angle^{(\mathrm{p})}$ tor $\epsilon_n^{(\mathrm{p})}$	$= \phi_n^{(p)}(m)$ $= \hbar\omega(m)$	$(q) = \left(rac{\lambda}{\pi} ight)$ $(1+rac{1}{2})$	$\int^{\frac{1}{4}} \left(2^n q_o\right)^{\frac{1}{4}} \left(2^n q_$	$(m!)^{-\frac{1}{2}}H_n(\sqrt{\lambda})$ $= c/m\omega^2$	$\left(q - \frac{c}{m\omega^2}\right) \epsilon$	$e^{-\frac{1}{2}\lambda(q-c/m\omega^2)^2}$
$\langle q' e^{-iH_r^{(s)}(t-t_o)/\hbar} q\rangle =$	$\left[\frac{m\omega}{2i\pi\hbar\sin\omega(t-t_0)}\right]$ $\exp\left\{\frac{im\omega}{2\hbar\sin\omega(t-t_0)}\right\}$	$\frac{1}{b} \int_{0}^{\frac{1}{2}} \times \frac{1}{t_{0}} \left[\left(q^{\prime 2} + \frac{1}{b} \right) \right]_{0}^{\frac{1}{2}} $	$+ q^2)\cos\omega(t)$	pr $(-t_0)$ -	opagato - 2q'q]	ors oscilla	tor
shifted	oscillator	$\langle q' e^{-iH_r^{(s)}}$	$^{(t-t_o)/\hbar} q angle$	$= \begin{bmatrix} \frac{1}{2\tau} \\ \exp \\ = i \omega ($	$\frac{\lambda}{\tau \sinh\xi} \Big]^{\frac{1}{2}} \times \Big\{ -\frac{\lambda}{4} \Big[(q' + (t - t_o)) \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \times \frac{1}{4} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \times \frac{1}{4} \Big[(q' + (t - t_o)) \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \times \frac{1}{4} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \times \frac{1}{4} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \times \frac{1}{4} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \times \frac{1}{4} \Big]^{\frac{1}{2}} \Big]^{\frac{1}{2}} \times \frac{1}{4} \Big]^{\frac{1}{2}} \Big]^{1$	$(q)^2 \tanh \frac{\xi}{2} + (q)^2 \tanh \frac{\xi}{2}$	$q' - q)^2 \operatorname{coth} \frac{\xi}{2} \bigg] \bigg]$

The reactant states (57) are occupied in thermal equilibrium with probability

$$p_n^{(r)} = x^n (1 - x), \quad x = e^{-\hbar\omega/kT},$$

a result which is well-known from elementary statistical mechanics. The corresponding equilibrium state density matrix of the reactant state oscillator $\rho_o^{(r)}$ has the matrix elements

$$\left[\hat{\rho}_o^{(\mathbf{r})}\right]_{mn} = p_n^{(\mathbf{r})} \,\delta_{nm} \,.$$

The density matrix can also be written

$$\hat{\rho}_o^{(\mathbf{r})} = 2 \sinh(\hbar \omega / 2kT) e^{-H^{(\mathbf{r})}/kT}$$

The transitions from reactant to product states are induced through the matrix elements v in (54). In case of electron transfer in proteins, the coupling is induced through electron tunneling between prosthetic groups in the protein. The corresponding energy values v are very small, usually of the order of 10^{-4} eV. As a result, reactant states $|n\rangle^{(r)}$ and product states $|m\rangle^{(p)}$ couple only when they are essentially degenerate. The overall rate of transfer from reactant states R to product states P

is then

$$k_{qo}(R \to P) = \frac{2\pi}{\hbar^2} v^2 \mathcal{S}_{qo}(E)$$

where

$$\mathcal{S}_{qo}(E) = \sum_{n,m=0}^{\infty} p_n^{(r)} |^{(r)} \langle n|m\rangle^{(p)} |^2 \delta\left(\frac{E + \epsilon_m^{(p)} - \epsilon_n^{(r)}}{\hbar}\right)$$

is the so-called spectral line shape function.

We seek to express the line shape function (70) in a more compact form. For this purpose we use the identity

$$\delta\left(\frac{E + \epsilon_m^{(p)} - \epsilon_n^{(r)}}{\hbar}\right) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \; e^{itE/\hbar} \; e^{-it\epsilon_n^{(r)}/\hbar} \; e^{it\epsilon_m^{(p)}/\hbar}$$

Employing the definition of the density matrix $\left[\hat{\rho}_{o}^{(\mathbf{r})}\right]_{mn} = p_{n}^{(\mathbf{r})} \delta_{nm}$ one can write

$$\mathcal{S}_{qo}(E) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \; e^{itE/\hbar} \; \sum_{n,m=0}^{\infty} {}^{(\mathbf{r})} \langle n | \; \hat{\rho}_o^{(\mathbf{r})} \; e^{-it\hat{H}_r^{(s)}/\hbar} \, |m\rangle^{(\mathbf{p})(\mathbf{p})} \langle m | \; e^{it\hat{H}_p^{(s)}/\hbar} \, |n\rangle^{(\mathbf{r})}$$

or, equivalently, using $\hat{\rho}_o^{(\mathbf{r})} = 2 \sinh(\hbar \omega / 2kT) e^{-H^{(\mathbf{r})}/kT}$

$$\mathcal{S}_{qo}(E) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \; e^{itE/\hbar} \; 2 \sinh \frac{\hbar\omega}{2kT} \operatorname{tr} \left(e^{-\hat{H}_r^{(s)}/kT} \; e^{-it\hat{H}_r^{(s)}/\hbar} \; e^{it\hat{H}_p^{(s)}/\hbar} \right)$$

Expressing the trace as an integral over q^\prime we conclude that the spectral line shape function is

$$S_{qo}(E) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \; e^{itE/\hbar} \; 2\sinh\frac{\hbar\omega}{2kT} \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dq' \langle q'| \; e^{-i(t-\hbar/kT)\hat{H}_r^{(s)}/\hbar} |q\rangle \langle q \; e^{it\hat{H}_p^{(s)}/\hbar} |q'\rangle$$

The propagator (64) allows one to evaluate the line shape function (74). One employs

$$\langle q | e^{it\hat{H}_{p}^{(s)}/\hbar} | q' \rangle = \left[\frac{\lambda}{2\pi \sinh\eta_{1}} \right]^{\frac{1}{2}} \times \\ \times \exp\left\{ -\frac{\lambda}{4} \left[(q' + q - 2q_{o})^{2} \tanh\frac{\eta_{1}}{2} + (q' - q)^{2} \coth\frac{\eta_{1}}{2} \right] \right\} \\ \eta_{1} = -i\omega t .$$

and, displacing time into the complex plane to account for the equilibrium (temperature T) density matrix,

$$\langle q'| e^{-i(t-\hbar/kT)\hat{H}_r^{(s)}/\hbar} |q\rangle = \left[\frac{\lambda}{2\pi\sinh\eta_2}\right]^{\frac{1}{2}} \times \\ \times \exp\left\{-\frac{\lambda}{4}\left[(q'+q)^2\tanh\frac{\eta_2}{2} + (q'-q)^2\coth\frac{\eta_2}{2}\right]\right\} \\ \eta_2 = i\omega t - \hbar\omega/kT .$$

Inserting (75-78) into (74) results in the expression

$$\mathcal{S}_{qo}(E) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \; e^{itE/\hbar} \; \frac{\lambda \sinh\left(\frac{\hbar\omega}{2kT}\right)}{\pi \sqrt{\sinh\eta_1 \, \sinh\eta_2}} \; I(t)$$

where

$$I(t) = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dq' \exp\left[-\alpha (t)(q+q')^2 - \beta (q+q'-2q_o)^2 - \gamma (q-q')^2\right]$$

$$\alpha = \frac{\lambda}{4} \tanh \frac{\eta_2}{2}$$

$$\beta = \frac{\lambda}{4} \tanh \frac{\eta_1}{2}$$

$$\gamma = \frac{\lambda}{4} \left(\tanh \frac{\eta_1}{2} + \tanh \frac{\eta_2}{2} \right)$$

Expression (79–83) for the spectral line shape function played an important role in the theory of spectral transitions of so-called F-centers in solids as reviewed in [21]. The expression can be further simplified [21]. For this purpose one transforms to new integration variables u = q + q' and u' = q - q'. Noting that for the Jacobian holds $\partial(u, u')/\partial(q, q') = 2$, the integral (80) reads

$$I(t) = \frac{1}{2} \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} du' \exp\left[-\alpha (t)u^2 - \beta (u - 2q_o)^2\right] \exp\left[-\gamma u'^2\right] .$$

Completion of the square in the first exponent results in the expression

$$I(t) = \frac{1}{2} \exp\left[-4q_o^2 \left(\beta - \frac{\beta^2}{\alpha + \beta}\right)\right] \times \\ \times \int_{-\infty}^{+\infty} du' \exp\left[-\gamma u'^2\right] \int_{-\infty}^{+\infty} du \exp\left[-(\alpha + \beta) (u - s)^2\right] .$$

where

$$s = 2\beta q_o / (\alpha + \beta) .$$

a few more steps of Gaussian integration ...

Combining Eqs. (62, 79, 87, 88, 91) results in

$$\mathcal{S}_{qo}(E) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \exp\left[itE/\hbar - \frac{c^2}{2m\hbar\omega^3} \coth\frac{\hbar\omega}{2kT}(1 - \cos\omega t) + i\frac{c^2}{2m\hbar\omega^3}\sin\omega t\right]$$

$$k_{qo}(R \to P) = \frac{2\pi}{\hbar^2} v^2 \mathcal{S}_{qo}(E)$$



The integral in (92) can be carried out and the line shape function expressed as a series of regular, modified Bessel functions $I_k(x)$ [21]. The result is

$$\mathcal{S}_{qo}(E) = \frac{e^{-\Lambda(1+2n_o)}}{\omega} \left(\frac{n_o+1}{n_o}\right)^{s_j/2} \sum_{k=-\infty}^{\infty} \delta\left(k-s(E)\right) \ I_k\left(2\Lambda\sqrt{n_o(n_o+1)}\right) \tag{94}$$

where $\Lambda = \frac{1}{2}m\omega^2 q_o^2/\hbar\omega = c^2/2m\hbar\omega^3$ is the so-called reorganization energy in units of vibrational quanta $\hbar\omega$, $n_o = e^{-\hbar\omega/kT}/(1-e^{-\hbar\omega_j/kT})$ is the average number of quanta thermally excited in the oscillator, and $s(E) = (E - \frac{1}{2}\hbar\omega)/\hbar\omega$ counts the number of oscillator levels up to energy E. The summation in (93) is over integers k such that one and only one term in the sum contributes anytime that s(E) assumes an integer value.

Electron Transfer in the Photosynthetic Reaction Center



Two State QM System Coupled to Protein Matrix

First case: Coupling to a single oscillator

Hamiltonian
$$\hat{H}_{qo}^{(s)} = \begin{pmatrix} \hat{H}_r^{(s)} & v \\ v & \hat{H}_p^{(s)} + E \end{pmatrix}$$

Protein matrix is a bath of oscillators linearly coupled to the electron transfer according to

$$\hat{H}_{r}^{(s)} = \frac{\hat{p}^{2}}{2m} + \frac{1}{2}m\omega^{2}q^{2}$$
$$\hat{H}_{p}^{(s)} = \frac{\hat{p}^{2}}{2m} + \frac{1}{2}m\omega^{2}\left(q - \frac{c}{m\omega^{2}}\right)^{2}$$
only a single oscillator considered at present



We start from the previously derived result

$$\mathcal{S}_{qo}(E) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \exp\left[itE/\hbar - \frac{c^2}{2m\hbar\omega^3} \coth\frac{\hbar\omega}{2kT} \left(1 - \cos\omega t\right) + i\frac{c^2}{2m\hbar\omega^3} \sin\omega t\right]$$
(92)

$$k_{qo}(R \to P) = \frac{2\pi}{\hbar^2} v^2 \mathcal{S}_{qo}(E)$$

Two State QM System Coupled to Protein Matrix

Second case: Coupling to an ensemble of oscillators

Hamiltonian
$$\hat{H}_{qo}^{(s)} = \begin{pmatrix} \hat{H}_r^{(s)} & v \\ v & \hat{H}_p^{(s)} + E \end{pmatrix}$$

Protein matrix is a bath of oscillators linearly coupled to the electron transfer according to

$$\hat{H}_r = \sum_j \left(\frac{\hat{p}_j^2}{2M_j} + \frac{1}{2} M_j \omega_j^2 q_j^2 \right)$$
$$\hat{H}_p = \sum_j \left(\frac{\hat{p}_j^2}{2M_j} + \frac{1}{2} M_j \omega_j^2 \left(q_j - \frac{c_j}{M_j \omega_j^2} \right)^2 \right)$$

Electron Transfer Process Coupled to the Protein Matrix

Rate for an ensemble of oscillators

$$k_{qb}(R \to P) = \frac{v^2}{\hbar^2} \int_{-\infty}^{+\infty} dt \; e^{itE/\hbar} \; e^{iQ_1(t)/\pi\hbar} \; e^{-Q_2(t)/\pi\hbar}$$

Relaxation rate

$$k_{\rm rel} = \frac{2v^2}{\hbar^2} \int_0^{+\infty} dt \cos(-itE/\hbar) \cos(Q_1(t)/\pi\hbar) e^{-Q_2(t)}$$
$$Q_1(t) = \frac{\pi}{2} \sum_j \frac{c_j^2}{\hbar\omega_j^3} \sin\omega_j t$$
$$Q_2(t) = \frac{\pi}{2} \sum_j \frac{c_j^2}{\hbar\omega_j^3} \coth\frac{\hbar\omega_j}{2kT} \left[1 - \cos(\omega_j t)\right]$$



But we don't know all the coupling constants c_i? All we need to know is

Solution of Rate Equation

$$\frac{d}{dt} \begin{pmatrix} p_R(t) \\ p_P(t) \end{pmatrix} = \begin{pmatrix} -k_{qb}(R \to P) & k_{qb}(P \to R) \\ k_{qb}(R \to P) & -k_{qb}(P \to R) \end{pmatrix} \begin{pmatrix} p_R(t) \\ p_P(t) \end{pmatrix}$$

$$\begin{pmatrix} p_R(t) \\ p_P(t) \end{pmatrix} = \frac{1}{k_{qb}(R \to P) + k_{qb}(P \to R)} \left[\begin{pmatrix} k_{qb}(P \to R) \\ k_{qb}(R \to P) \end{pmatrix} + \left(\frac{k_{qb}(R \to P)}{-k_{qb}(R \to P)} \right) \exp\left\{ - \left[k_{qb}(R \to P) + k_{qb}(P \to R) \right] \right\}$$

Electron Transfer Process Coupled to the Protein Matrix

Relaxation rate

$$k_{\rm rel} = \frac{2v^2}{\hbar^2} \int_0^{+\infty} dt \cos(-itE/\hbar) \cos(Q_1(t)/\pi\hbar) e^{-Q_2(t)/\pi\hbar}$$

$$Q_1(t) = \int_0^{\infty} d\omega \, \omega^{-2} J(\omega) \sin\omega t$$

$$Q_2(t) = \frac{\pi}{2} \int_0^{\infty} d\omega \, \omega^{-2} J(\omega) \coth\frac{\hbar\omega}{2kT} (1 - \cos\omega t)$$

$$\frac{J(\omega)}{\omega} = \frac{\sigma^2}{k_B T} \int_0^{\infty} dt \, C(t) \cos\omega t$$

$$C_{\epsilon\epsilon}(t) = \frac{\langle (\epsilon(t) - \langle \epsilon \rangle) (\langle \epsilon(0) - \langle \epsilon \rangle) \rangle}{\langle \epsilon(0) - \langle \epsilon \rangle \rangle^2}$$
energy gap correlation function
$$\sigma \quad \text{rms deviation of energy gap}$$





Temperature Dependence of Electron Transfer Rate



Classical (High Temperature) Limit of Electron Transfer



redox energy & (kcal/mol)

Figure 2: Comparison of electron transfer rates $k(\epsilon, T)$ shown as a function of ϵ evaluated in the framework of the spin-boson model (solid lines) and by Marcus theory (dashed lines) at temperatures 10 K and 300 K. The functions are centered approximately around ϵ_m . From [3].