Notes on Quantum Mechanics

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Preface

The following notes introduce *Quantum Mechanics* at an advanced level addressing students of Physics, Mathematics, Chemistry and Electrical Engineering. The aim is to put mathematical concepts and techniques like the path integral, algebraic techniques, Lie algebras and representation theory at the readers disposal. For this purpose we attempt to motivate the various physical and mathematical concepts as well as provide detailed derivations and complete sample calculations. We have made every effort to include in the derivations all assumptions and all mathematical steps implied, avoiding omission of supposedly 'trivial' information. Much of the author's writing effort went into a web of cross references accompanying the mathematical derivations such that the intelligent and diligent reader should be able to follow the text with relative ease, in particular, also when mathematically difficult material is presented. In fact, the author's driving force has been his desire to pave the reader's way into territories unchartered previously in most introductory textbooks, since few practitioners feel obliged to ease access to their field. Also the author embraced enthusiastically the potential of the TEX typesetting language to enhance the presentation of equations as to make the logical pattern behind the mathematics as transparent as possible. Any suggestion to improve the text in the respects mentioned are most welcome. It is obvious, that even though these notes attempt to serve the reader as much as was possible for the author, the main effort to follow the text and to master the material is left to the reader.

The notes start out in Section 1 with a brief review of *Classical Mechanics* in the Lagrange formulation and build on this to introduce in Section 2 *Quantum Mechanics* in the closely related *path integral formulation*. In Section 3 the *Schrödinger equation* is derived and used as an alternative description of continuous quantum systems. Section 4 is devoted to a *detailed presentation of the harmonic oscillator*, introducing algebraic techniques and comparing their use with more conventional mathematical procedures. In Section 5 we introduce the *presentation theory of the 3-dimensional rotation group and the group SU(2)* presenting Lie algebra and Lie group techniques and applying the methods to the theory of angular momentum, of the spin of single particles and of angular momenta and spins of composite systems. In Section 6 we present the *theory of many–boson and many–fermion systems* in a formulation exploiting the algebra of the associated creation and annihilation operators. Section 7 provides an introduction to *Relativistic Quantum Mechanics* which builds on the representation theory of the Lorentz group and its complex relative $Sl(2, \mathbb{C})$. This section makes a strong effort to introduce Lorentz–invariant field equations systematically, rather than relying mainly on a heuristic amalgam of Classical Special Relativity and Quantum Mechanics.

The notes are in a stage of continuing development, various sections, e.g., on the semiclassical approximation, on the Hilbert space structure of Quantum Mechanics, on scattering theory, on perturbation theory, on Stochastic Quantum Mechanics, and on the group theory of elementary particles will be added as well as the existing sections expanded. However, at the present stage the notes, for the topics covered, should be complete enough to serve the reader.

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These notes were produced entirely on a Macintosh II computer using the TEX typesetting system, Textures, Mathematica and Adobe Illustrator.

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