Extra Credit Problem for Problem Set 10
Physics 480 / Fall 1999
Professor Klaus Schulten

Problem: Hydrogen Atom in External Electric Field (Stark Effect in Hydrogen)

Consider a single electron in the $n = 2$ state of the hydrogen atom. We ignore relativistic corrections, so the $2s$ and $2p$ states are initially degenerate. It is convenient to denote these four orbital states by $|\ell\, m\rangle$, where $\ell = 0, 1$, and $m = 0, \ldots, \pm\ell$. When a small static electric field $\mathbf{E} = E\hat{\mathbf{z}}$ is imposed, the energy levels of the atom will change (*Stark effect*). By using perturbation theory, you are asked to derive how the $n = 2$ energy levels are changed to lowest order in powers of E.

According to perturbation theory, the lowest order correction to the energies of the degenerate $n = 2$ states are given by the eigenvalues of the perturbing potential $\hat{V} = -e\mathbf{E}\mathbf{r}$ in the Hilbert space spanned by the four states $|\ell\, m\rangle$.

(a) By invoking parity conservation (or otherwise) show that the only non-vanishing matrix elements for $\hat{V}$ are

$$\varepsilon = \langle 0\,0|\hat{V}|1\,0\rangle = \langle 10|\hat{V}|0\,0\rangle\ .$$

(b) Calculate explicitly the value of ε.

(c) Write down the energies and wave functions (as a linear combination of $|\ell\, m\rangle$) corresponding to the perturbed $n = 2$ states.

The problem set needs to be handed in by Tuesday, December 2. The web page of Physics 480 is at
`http://www.ks.uiuc.edu/Services/Class/PHYS480/`