Problem Set 9
Physics 386 / Spring 1997
Professor Klaus Schulten

Problem 1: Goswami, Chapter 6, Problem 4

Prove that for any operator A, $A + A^\dagger$ and $i(A - A^\dagger)$ are hermitian.

Problem 2: Goswami, Chapter 6, Problem 5

Prove that if A is a hermitian operator, $\langle A^2 \rangle \geq 0$.

Problem 3 : Goswami, Chapter 6, Problem A7

An arbitrary unitary operator U can be decomposed as

$$U = \frac{U + U^\dagger}{2} + i\frac{U - U^\dagger}{2i} \equiv H_1 + iH_2 \tag{1}$$

(a) Show that H_1 and H_2 are hermitian operators.
(b) Show $[H_1, H_2] = [U, H_1] = [U, H_2] = 0$; thus U, H_1, and H_2 have simultaneous eigenstates. Call these $|h_1 h_2\rangle$. Using these eigenstates, show that
(c) the eigenvalues of U are of unit magnitude.

Problem 4: Goswami, Chapter 7, Problem A8, 9

The operator c is defined by the following relations:

$$\begin{aligned} c^2 &= 0 & (2) \\ cc^\dagger + c^\dagger c &= 1 & (3) \end{aligned}$$

where $c^\dagger$ is the hermitian conjugate of c. Show that
(a) $N = c^\dagger c$ is hermitian.
(b) $N^2 = N$.
(c) The eigenvalues of N are 0 and 1.
(d) If $|0\rangle$ and $|1\rangle$ denote the two eigenstates of N corresponding to the eigenvalues 0 and 1, respectively, show that

$$\begin{aligned} c^\dagger |0\rangle &= |1\rangle & (4) \\ c\,|0\rangle &= 0. & (5) \end{aligned}$$

Consider the Hamiltonian

$$H = \hbar\omega_0(c^\dagger c + \frac{1}{2}) \tag{6}$$

where c, $c^\dagger$ are anticommuting as above. Denoting the eigenstates of H as $|n\rangle$, show that the only nonvanishing states are the states $|0\rangle$ and $|1\rangle$ above.

This problem set needs to be handed in by Wednesday, April 30, 1997.