Problem Set 3
Physics 386 / Spring 1997
Professor Klaus Schulten

Problem 1: Fourier Transform

Find the Fourier transform of the following function

$$\psi(x) \;=\; \begin{cases} 1 \,-\, |x| & \text{for } |x| < 1 \\ 0 & \text{for } |x| \geq 1 \end{cases} \tag{1}$$

Problem 2: Expectation Values in Momentum Space

For the expectation value in momentum space defined through

$$\langle f(k) \rangle \;=\; \frac{\int_{-\infty}^{+\infty} a^*(k)\, f(k)\, a(k)}{\int_{-\infty}^{+\infty} a^*(k)\, a(k)} \tag{2}$$

show that for

$$a(k) \;=\; \exp\left(-\frac{(k \,-\, k_0)^2}{2(\delta k)^2} \right) \tag{3}$$

and

$$\Delta k \;=\; \sqrt{\langle k^2 \rangle \,-\, \langle k \rangle^2} \tag{4}$$

follows

$$\Delta p \;=\; \hbar \Delta k \;=\; \frac{\hbar \delta k}{\sqrt{2}}\,. \tag{5}$$

Problem 3: Ground State of the Harmonic Oscillator

Use the uncertainty relationship to find an estimate of the ground state energy of a harmonic oscillator. The energy of the oscillator is

$$E \;=\; \frac{p^2}{2m} \,+\, \frac{1}{2} m\,\omega^2 x^2 \tag{6}$$

where ω is the oscillator frequency.

Problem 4: Life Time Broadening

An excited state of an atom has a life time of 10^{-9} s. What is the natural line width of the energy level?

This problem set needs to be handed in by Monday, February 24, 1997.