Problem Set 10
Physics 386 / Spring 1997
Professor Klaus Schulten

Problem 1: Avoided Crossing

The Hamiltonian matrix for a two-state quantum system is given by

$$\hat{H}(p) = \begin{pmatrix} \epsilon p & v \\ v & -\epsilon p \end{pmatrix} \qquad (1)$$

where p is a real parameter and ϵ, v are real constants.

(a) Evaluate and plot the energy eigenvalues (in units of v) for $p \in [-p_0, +p_0]$ where $p_0 = 5v/\epsilon$.

(b) Let $|\tilde{1}\rangle$ be the eigenvector of (1) corresponding to a positive eigenvalue. Plot for the same range of p-values the two components of the eigenvector. Do the same for the eigenvector corresponding to negative eigenvalues.

Problem 2: Four State Quantum System

A four state quantum system is described by the Hamiltonian

$$\hat{H} = \begin{pmatrix} a & 0 & 0 & -b \\ 0 & c & id & 0 \\ 0 & -id & -c & 0 \\ -b & 0 & 0 & -a \end{pmatrix} \qquad (2)$$

where $i = \sqrt{-1}$.

(a) Find the eigenvalues and the eigenvectors of this Hamiltonian (Hint: If you rearrange the ordering of the basis, you can find an ordering in which the matrix becomes block-diagonal.)

(b) Assume that the system is initially in a state

$$|\psi(0)\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad (3)$$

in the basis corresponding to (2) (i.e., *not* an eigenstate). Determine $|\psi(t)\rangle$.

Problem 3: Displaced Harmonic Oscillator in Matrix Representation

(a) Determine by means of the creation and annihilation operators $\hat{a}^+$ and $\hat{a}$ introduced in class the matrix representation of the Hamiltonian $H_a = H_o + m\omega^2 a x$ in the basis of eigenstates $|n\rangle_o$ of the undisplaced oscillator $H_o = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$, i.e., $H_o|n\rangle_o = \hbar\omega(n+\frac{1}{2})|n\rangle_o$.

(b) Provide a simple argument why the energy of the ground state $|0\rangle_a$ is $E_{oa} = \hbar\omega(\frac{1}{2} - \frac{m\omega}{2\hbar}a^2)$.

(c) Expand the state $|0\rangle_a$ in terms of eigenstates of the undisplaced oscillator

$$|0\rangle_a = \sum_{n=0}^{\infty} c_n |n\rangle_o \tag{4}$$

and show that the coefficients c_n obey the eigenvalue problem

$$\sum_{n=0}^{\infty} (\, H_{mn} - E_{oa}\,\delta_{mn}\,)\, c_n = 0\,, \tag{5}$$

where

$$H_{mn} = {}_o\langle m|H_a|n\rangle_o \tag{6}$$

and E_{oa} have been determined in (a) and (b).

(d) Express c_m in terms of c_0 and determine c_o though the normalization condition ${}_a\langle 0|0\rangle_a = 1$. Sketch c_n^2 as a function of n for various a values.

This problem set needs to be handed in by Wednesday, May 7, 1997.